Suppose

Program 1 recognizes L_1

& Program 2 recognizes L_2

Is there a program recognizing

$L_1 \cup L_2$?

$L_1 \cap L_2$?

$\vdots$

Closure under $\cup$

even parity $\subseteq \{0,1\}^*$

or 3rd from right is a $\subseteq \{a,b\}^*$

... - - - - - - 1 $\subseteq \{0,1\}^*$

$M_i = (Q_i, \Sigma, \delta_i, q_{0i}, F_i)$

$M = (Q_1 \times Q_2, \Sigma, \delta, (q_{01}, q_{02}), F)$

$\forall\ q_1 \in Q_1, q_2 \in Q_2\ \ a \in \Sigma$

$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$

Equivalent

$F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$

$F = \{ (a, b) \mid \text{either } a \in F_1 \text{ or } b \in F_2 \}$

Claim:

$\forall q_1 \in Q_1, \forall q_2 \in Q_2, \forall w \in \Sigma^*$

M is in state (q_1, q_2) after reading w $\Leftrightarrow$ M_1 is in q_1 after reading w and M_2 is in q_2 .. .. ..

Proof:

Homework (induction on $|w|$)

Corollary:

$L(M) = L(M_1) \cup L(M_2)$

Note:

Claim looks a lot like defn of δ.

BUT $\delta(-, a)$ for finite set $a \in \Sigma$

claim "... w" for infinite set $w \in \Sigma^*$

$X, Y \subseteq \Sigma^*$

$X \cdot Y = \{x \cdot y \mid x \in X \,\&\, y \in Y\}$

Examples

$L_{oddparity} \cdot L_{oddparity} = L_{even} - \{0\}^*$

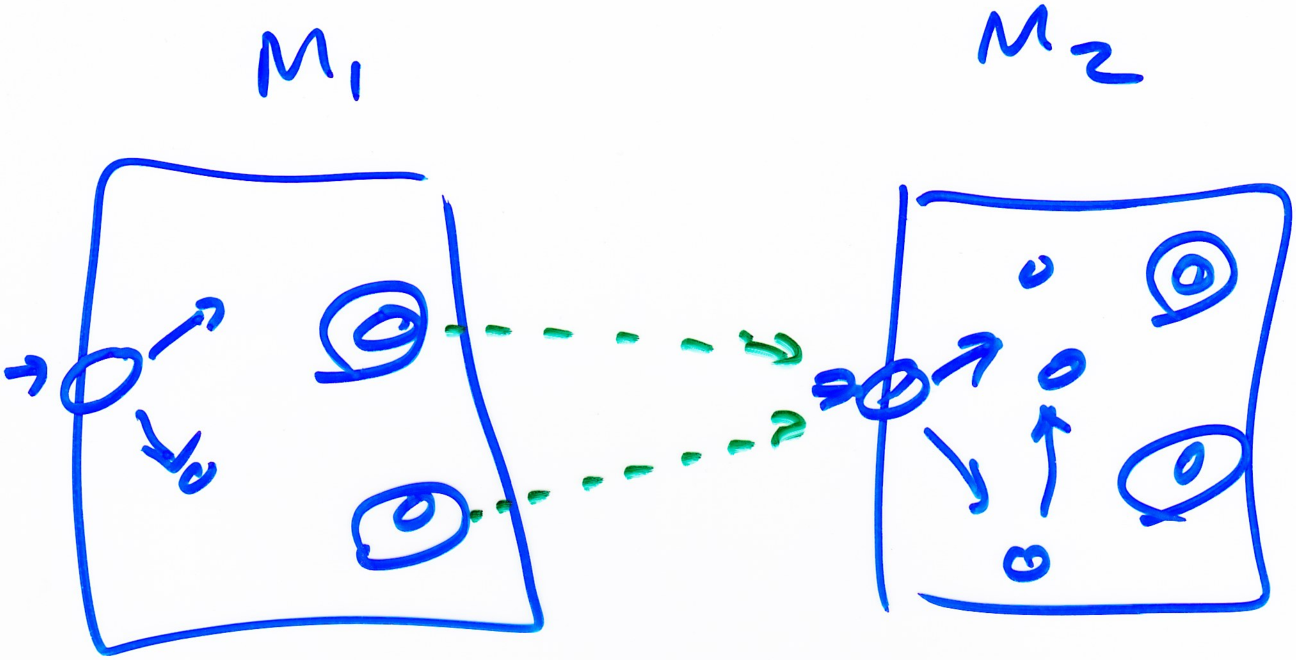
$L_{odd\,parity} \cdot L_{even} = L_{odd}$

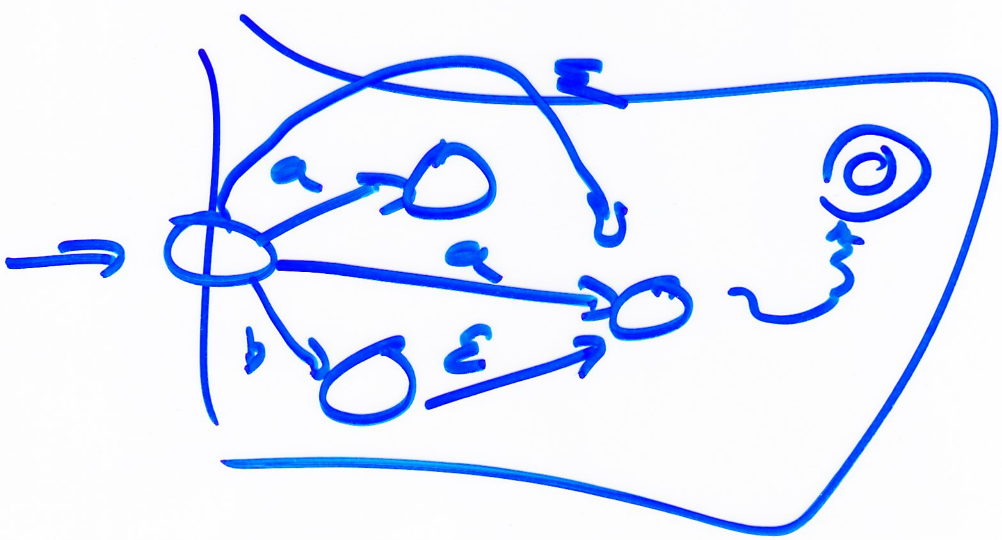
$\underset{\text{infinite}}{A} \cdot \underset{\text{finite}}{B} \overset{?}{=} \underset{\text{Finite}}{C}$] possible?

$\Sigma^* \cdot \phi = \phi$] yes

$X \cdot Y \overset{?}{=} Y \cdot X$] always true?

$\{0\} \cdot \{1\} \neq \{1\} \cdot \{0\}$] no





{a, b, c}