CSE 322
Winter Quarter 2009
Assignment 7
Due Friday, February 20, 2009

All solutions should be neatly written or type set. All major steps in proofs must be justified. Please start each problem solution on a new page and put your name on every page.

1. (10 points) Given context-free grammars $G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$, design context-free grammars G such that:

   (a) $L(G) = L(G_1)L(G_2)$ (concatenation),

   (b) $L(G) = L(G_1)^*$ (Kleene star),

   (c) $L(G) = L(G_1)^R$ (reversal).

2. (10 points) Consider the context free-grammar:

$$\begin{aligned} S &\rightarrow ASAS \mid A \mid \varepsilon \\ A &\rightarrow 110 \mid \varepsilon \end{aligned}$$

   Use a method similar to that described in class to convert the grammar into Chomsky normal form. In this method do the steps in the following order: (i) add a new start symbol if ε is generated by the grammar, (ii) shorten productions whose right hand sides are longer than 2, (iii) remove ε-rules, (iv) remove unit rules, (v) make all right hand sides of length 2 into nonterminals (This step can be done earlier, but it doesn't change the efficiency of the algorithm).

3. (10 points) Let $G = (V, \Sigma, R, S)$.

   (a) A nonterminal A is *productive* if $A \Rightarrow_G^* w$ for some $w \in \Sigma^*$. That is, some terminal string can be generated from A. Design a closure algorithm for finding all the productive nonterminals in a grammar G. As a hint, initialize the set $X = \{A : A \rightarrow x \text{ is a production for some } x \in \Sigma^*\}$. The closure algorithm may then add more nonterminals to X as it proceeds.

   (b) Use the algorithm in part (a) as part of an algorithm for deciding if the language generated by a context-free grammar is empty.