CSE 322
Winter Quarter 2009
Assignment 2
Due Friday, January 16, 2009

All solutions should be neatly written or type set. All major steps in proofs must be justified. Please start each problem solution on a new page and put your name on every page.

1. (10 points) This problem is designed to strengthen your ability to prove facts by induction. The reversal w^R of a string w can be defined recursively in the following way.

$$\begin{aligned} \varepsilon^R &= \varepsilon \\ (xa)^R &= ax^R \end{aligned}$$

   where $a \in \Sigma$.

   Prove the following: For all strings x and y over Σ, $(xy)^R = y^R x^R$. For this your proof should be by induction on the length of y. You may use recursive definition of reversal and any basic facts about concatenations such as associativity and the identity properties of ε. That is: $x(yz) = (xy)z$ and $\varepsilon x = x\varepsilon = x$ for all strings x, y, z.

2. (10 points) Design deterministic finite automata using a state transition diagram for each of the following languages.

   (a) $\{x \in \{0,1\}^* \ : \ 101$ is a substring of $x\}$.

   (b) $\{x \in \{0,1\}^* \ : \ 111$ is not a substring of $x\}$.

   (c) $\{x \in \{0,1\}^* \ : \ x$ contains exactly 5 $0's\}$.

   (d) $\{x \in \{0,1\}^* \ : \ x$ has an odd number of $0's$ or an even number of $1's\}$.

3. (10 points) Consider the languages

$$L_k = \{x \in \{0,1\}^* \ : \ x \text{ contains exactly } k \ 0's\}$$

   for $k \geq 0$.

(a) Formally define a deterministic finite automaton M_k with exactly $k + 2$ states that accepts L_k.

(b) Prove by contradiction that every deterministic finite automaton that accepts L_k has at least $k + 2$ states. The ideas from problem 1 of assignment 1 are useful.