CSE 322
Winter Quarter 2009
Assignment 1
Due Friday, January 9, 2009

All solutions should be neatly written or type set. All major steps in proofs must be justified. Please start each problem solution on a new page and put your name on every page.

1. (5 points) This problem involves an application of the Pigeon Hole Principle. Simply stated the Pigeon Hold Principles states that if there are are m items placed in n bins and $m > n$, then some bin contains at least 2 items.

   Suppose we have a directed graph $G = (V, E)$ where V has n vertices. A path of length m in G is a sequence of vertices $x_0, v_1, \ldots, x_m$ where (x_i, x_{i+1}) is an edge for $0 \leq i < m$. The vertices $x_0, x_1, \ldots, x_m$ are *visited* by the path. Note that the number of edges on the path is its length. Use the Pigeon Hole Principle to prove that any path in G of length $\geq n$ visits some vertex at least twice.

2. (5 points) This problem involves a practice doing an induction proof.

   Suppose we have an alphabet of $m > 0$ symbols. Show by induction that the number of strings of length n in these symbols is m^n. Note that there is only one string of length 0, namely the empty string.

3. (5 points) This problem involves doing a proof by contradiction.

   Prove that the $\sqrt{3}$ is not a rational number. Recall that rational numbers are those that can be represented by fractions.