

**Reading Assignment:** Sipser 2.1

1. Design context-free grammars which generate each of the following languages. Justify your grammar designs. You may specify your CFG by writing out the rules with the convention that the first state is the start state. You may also use the “|” shorthand.
  - (a) The set  $\{w \in \{0, 1\}^* \mid w = w^R\}$ , i.e. the set of palindromic binary strings.
  - (b) The complement of the set  $\{a^n b^n \mid n \geq 0\}$  in  $\{a, b\}^*$ .
  - (c) The set  $\{w \in \{0, 1\}^* \mid \text{the length of } w \text{ is odd and its middle symbol is } 1\}$ .

2. Consider the grammar:

$$S \rightarrow (S) \mid SS \mid \varepsilon$$

- (a) Show that this grammar is ambiguous.
  - (b) Give a new unambiguous grammar for the same language.
3. Convert the following CFG into an equivalent CFG in Chomsky normal form:

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \varepsilon \\ B &\rightarrow 11 \mid \varepsilon \end{aligned}$$

4. **Extra credit:** Do it for the glory not the points. Let the alphabet be  $\Sigma = \{a, b\}$ . Give a CFG generating the language of strings with twice as many *as* as *bs*. Prove that your grammar is correct.