CSE 322: Formal Models in Computer Science **Homework #4**
April 24, 2009 Due: **Friday**, May 1, 2009

Reading Assignment: Lecture notes on pattern matching, Myhill-Nerode, and DFA Minimization. Sipser 2.1

Problems:

1. Use the pumping lemma to prove that the following languages are not regular:
   (a) $L_1 = \{ww | w \in \{a,b\}^*\}$.
   (b) $L_2 = \{0^n 1^m 0^n | m, n \geq 0\}$.
   (c) $L_3 = \{0^p | p \text{ is a prime number}\}$.

2. Use the method from the Myhill-Nerode theorem (see lecture notes) to prove that the following languages are not regular:
   (a) $L_4 = \{0^n 1^m 0^n | m, n \geq 0\}$.
   (b) $L_5 = \{w | w \neq w^R, w \in \{0,1\}^*\}$. Recall w^R is the reversal of the string w. So this is the language of strings which are not palindromes.

3. Show that the language

$$L_6 = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

   satisfies the conditions of the pumping lemma and therefore cannot be proven non-regular by the pumping lemma. That is show that the clause after the "then" in the pumping lemma can be satisfied for this language. Then use Myhill-Nerode (see lecture notes) to prove that the language is not regular.

4. Consider the language A of strings in $\{a,b\}^*$ that start and end in different symbols. Give the equivalence classes of this language under the equivalence relation of the language. Recall that two strings x and y are equivalent under A (which we write as $x \equiv_A y$) if for all strings $z \in \Sigma^*$ the proposition "$xz \in A$ if and only if $yz \in A$" holds. Here the alphabet is $\Sigma = \{a,b\}$.

5. **Extra Credit** Do it for the glory, not for the points! Let $C_k = \Sigma^* a \Sigma^{k-1}$ where $\Sigma = \{a,b\}$. Prove that a DFA which recognizes C_k must have 2^k states.