CSE 322: Formal Models in Computer Science | **Homework #1**
April 1, 2009 | Due: **Friday**, April 10, 2008

Reading Assignment: 0.1-0.4 (review) and 1.1-1.2

Problems:

1. The rule for valid names for variables in C programs is the following. Variables must begin with a character (that is, a letter in the English alphabet) or underscore and may be followed by any combination of characters, underscores, or the digits $0 - 9$. Design a DFA that accepts only strings that are valid variable names (For simplicity assume that $\Sigma = \{< c >, < d >, < u >, \#\}$ where $< c >$ denotes a character, $< d >$ denotes a digit, and $< u >$ denotes and underscore, and $\#$ denotes any other possible ASCII character. Express your answer both in the form of a state diagram **and** also formally by specifying the 5-tupe $(Q, \Sigma, \delta, q_0, F)$.

2. Give state diagrams of DFAs recognizing the following languages. In each parts assume that the alphabet is $\Sigma = \{0, 1\}$. As documentation for your DFA, for each state, give a description of the strings which will *end* at that given state. This means that for each state you should give a description of the set of strings which, if they were inputed to the DFA, would end at that state.

   (a) $L_1 =\{\ w \mid w$ contains at least two 1s and at least three 0s. $\}$

   (b) $L_2 =\{\ w \mid w$ has length at least 2 and its second symbol is a 0. $\}$

   (c) $L_3 =\{\ w \mid w$ has an even number of 0s and an odd number of 1s. $\}$

   (d) $L_4 =\{\ w \mid w$ begins with a 1, and which, interpreted as the binary representation of a positive integer, is divisible by 4 $\}$.
   For this last part assume that the DFA starts reading the string from its most signicant bit. For example if $w = 1000$, then w is the binary representation of the (decimal) number 8 (and thus, is in the language). and the DFA starts by reading the bit 1.

3. The reversal of a string w denoted by w^R, is the string when you look at it backwards: for example, $homer^R = remoh$. Here is the formal inductive definition (where the alphabet is Σ):

   Definition of reversal of a string
   Base case. If $w = \epsilon$, then $w^R = \epsilon$.
   Inductive step. If $w = va$ for $v \in \Sigma^*$ and $a \in \Sigma$, then $w^R = av^R$.

   Note in this definition that a is a single element of the alphabet.

   Prove by induction (on the length of y) that for all strings $x, y \in \Sigma^*$,

   $$(xy)^R = y^R x^R$$

Note that you will be doing an inductive proof of the above fact using the inductive definition. I.e. your prove will be inductive and as part of your proof you may use the definition (which is inductive) one or more times.

4. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In each part the alphabet is $\Sigma = \{0, 1\}$.

   (a) The language $\{0\}^*$ with one state. Recall $\{0\}^* = \{\varepsilon, 0, 00, 000, \dots\}$.

   (b) The language $\{\varepsilon\}$ with one state.

   (c) The language $\{w | w = x010y \text{ for some } x, y \in \{0, 1\}^*\}$ with four states.

5. **Extra Credit** (Minimal points, kept track of separately from main homework points. Do it for the glory and for the challenge!) Let

$$B_n = \{a^k | \text{where } k \text{ is a multiple of } n\}.$$

   Show that for each $n \geq 1$, the language B_n is regular.