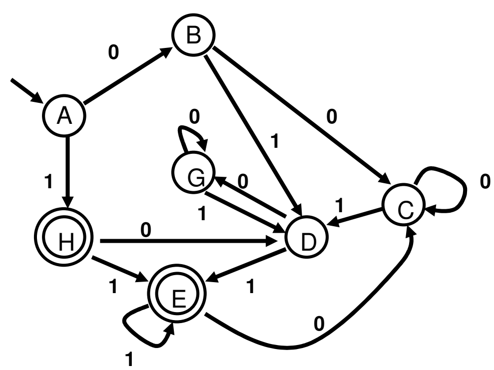
CSE 322 Autumn 2009
Assignment #5

Due: Friday, November 13, 2009 in class

Reading assignment: Read Section 2.1 of Sipser's book and the handout on Chomsky Normal Form.

Problems:

1. Apply the state minimization algorithm to the DFA below. Show each of your steps as in the example on the minimization handout.



2. Design context-free grammars that generate each of the following languages. Justify your grammar designs.

   (a) The set $\{w \in \{0,1\}^* \mid w = w^R\}$.

   (b) The complement of the language $\{a^n b^n \mid n \geq 0\}$ in $\{a,b\}^*$.

   (c) The set $\{w \in \{0,1\}^* \mid w$ contains twice as many 1's as 0's$\}$.

3. Sipser's text, 2nd edition Problem 2.16 (1st edition Problem 2.15).

4. In class we gave the following grammar

$$S \rightarrow (S) \mid SS \mid \varepsilon$$

   for the set of strings of balanced parentheses.

   (a) Show that this grammar is ambiguous.

   (b) Give a new unambiguous grammar for the same language.

5. Let $G = (V, \Sigma, R, \langle\text{STMT}\rangle)$ be the following grammar:

$$\begin{array}{rcl} \langle\text{STMT}\rangle & \rightarrow & \langle\text{ASSIGN}\rangle \mid \langle\text{IF-THEN}\rangle \mid \langle\text{IF-THEN-ELSE}\rangle \\ \langle\text{IF-THEN}\rangle & \rightarrow & \texttt{if condition then } \langle\text{STMT}\rangle \\ \langle\text{IF-THEN-ELSE}\rangle & \rightarrow & \texttt{if condition then } \langle\text{STMT}\rangle \texttt{ else } \langle\text{STMT}\rangle \\ \langle\text{ASSIGN}\rangle & \rightarrow & \texttt{a := 1} \end{array}$$

$$\begin{array}{rcl} \Sigma & = & \{\texttt{i}, \texttt{f}, \texttt{c}, \texttt{o}, \texttt{n}, \texttt{d}, \texttt{t}, \texttt{h}, \texttt{e}, \texttt{l}, \texttt{s}, \texttt{a}, \ , :, =, \texttt{1}\} \\ V & = & \{\langle\text{ASSIGN}\rangle, \langle\text{STMT}\rangle, \langle\text{IF-THEN}\rangle, \langle\text{IF-THEN-ELSE}\rangle, \langle\text{ASSIGN}\rangle\} \end{array}$$

G is a natural-looking grammar for a fragment of a programming language, but G is ambiguous.

(a) Show that G is ambiguous.

(b) Give a new unambiguous grammar for $L(G)$.

6. (Extra credit) A CFG $G = (V, \Sigma, R, S)$ is *regular* (also known as *right-linear*) iff every rule of G is of the form $A \rightarrow wB$ or $A \rightarrow w$ for $w \in \Sigma^*$ and $A, B \in V$. In class we showed that every regular language has a regular grammar. Show the converse, namely that for every regular grammar G, $L(G)$ is regular, which together with what we showed in class shows that regular grammars generate precisely the regular languages.

7. (Extra credit) Sipser's text, 2nd edition Problem 2.19 (1st edition Problem 2.25).