CSE 322 Winter 2008
Assignment #2

Due: Friday, January 25, 2008

Reading assignment: Finish reading sections 1.1-1.3 of Sipser's book.

Problems:

1. For languages A and B over alphabet Σ, let the *perfect shuffle* of A and B be the language

   $\{w \mid$ there is some $k \geq 0$ such that $w = a_1 b_1 \ldots a_k b_k$ where $a_1 \ldots a_k \in A$ and $b_1 \ldots b_k \in B$, each $a_i, b_i \in \Sigma.\}$

   (That is, it consists of all strings built by taking two strings of equal length from A and B and interleaving them as if they were cards in a perfect shuffle.) Given DFAs that recognize A and B give a brief intuitive description and then a formal description of how to build a DFA that recognizes the perfect shuffle of A and B.

2. Sipser's book 2nd edition Problem 1.34 (1st edition Problem 1.27). Document the states of your DFA.

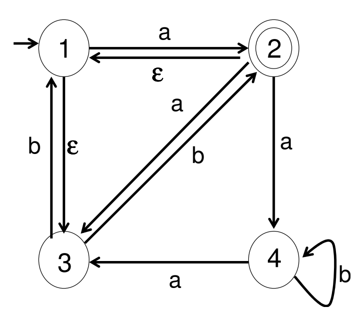
3. Draw NFAs with at most 8 states that recognize each of the following languages. Explain why each of your NFAs is correct. (Full state-by-state documentation may be used as part of this explanation but is not required.)

   (a) The set of all binary strings containing 0110 or 101.

   (b) The set of all binary strings other than 010 or 101.

4. Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let C_k be the language consisting of all strings that contain an 'a' exactly k places from the right-hand end. Thus $C_k = \Sigma^* a \Sigma^{k-1}$. Describe an NFA with $k + 1$ states that recognizes C_k, both in terms of a state diagram and a formal description.

5. Apply the subset construction to convert the following NFA to a DFA. Only the states reachable from the start state need to be shown.



6. **(Extra credit)** Sipser's book 2nd edition Problem 1.32 (1st edition Problem 1.25). Document the states of your DFA.

7. **(Extra credit** due Jan 26) Show that if A is recognized by a finite automaton there is a finite automaton that recognizes the set $A_{\frac{1}{2}-}$ of first halves of strings in A, i.e.

$$A_{\frac{1}{2}-} = \{x \; : \; xy \in A \text{ for some } y \text{ with } |x| = |y|\}.$$