CSE 322: Formal Models in Computer Science — **Assignment #1**

September 26, 2008 — Due: **Monday**, October 6, 2008

Reading Assignment: 0.1-0.4 (review) and 1.1-1.2

Problems:

1. Sipser's book, Exercise 1.3 (same in both editions.) Make sure you include everything that a state diagram should include!

2. The rule for valid names for variables in C programs is the following. Variables must begin with a character (that is, a letter in the English alphabet) or underscore and may be followed by any combination of characters, underscores, or the digits $0-9$. Design a DFA that accepts strings that are valid variable names (For simplicity assume that $\Sigma = \{< c >, < d >, < u >, \#\}$ where $< c >$ denotes a character, $< d >$ denotes a digit, and $< u >$ denotes and underscore, and # denotes any other possible ASCII character.

3. Give state diagrams of DFAs recognizing the following languages. In each parts assume that the alphabet is $\Sigma = \{0, 1\}$. As documentation for your DFA, for each state, give a description of the strings which will *end* at that given state. This means that for each state you should give a description of the set of strings which, if they were inputed to the DFA, would end at that state.

   (a) $L_1 =\{\ w \mid w$ contains at least three 1s and at least three 0s. $\}$

   (b) $L_2 =\{\ w \mid w$ has length at least 2 and its second symbol is a 0. $\}$

   (c) $L_3 =\{\ w \mid w$ has an even number of 0s and an odd number of 1s. $\}$

   (d) $L_4 =\{\ w \mid w$ begins with a 1, and which, interpreted as the binary representation of a positive integer, is divisible by 4 $\}$.
   For this last part assume that the DFA starts reading the string from its most signicant bit. For example if $w = 1000$, then w is the binary representation of the (decimal) number 8 (and thus, is in the language). and the DFA starts by reading the bit 1.

4. The reversal of a string w denoted by w^R, is the string when you look at it backwards: for example, $homer^R = remoh$. Here is the formal inductive definition (where the alphabet is Σ):

   Definition of reversal of a string
   Base case. If $w = \epsilon$, then $w^R = \epsilon$.
   Inductive step. If $w = va$ for $v \in \Sigma^*$ and $a \in \Sigma$, then $w^R = av^R$.

   Note in this definition that a is a single element of the alphabet.

Prove by induction (on the length of y) that for all strings $x, y \in \Sigma^*$,

$$(xy)^R = y^R x^R$$

Note that you will be doing an inductive proof of the above fact using the inductive definition.

5. **Extra Credit** (minimal points, do it for the glory!) Sipser's book, Problem 1.37 in second edition (Problem 1.30 in the first edition.)