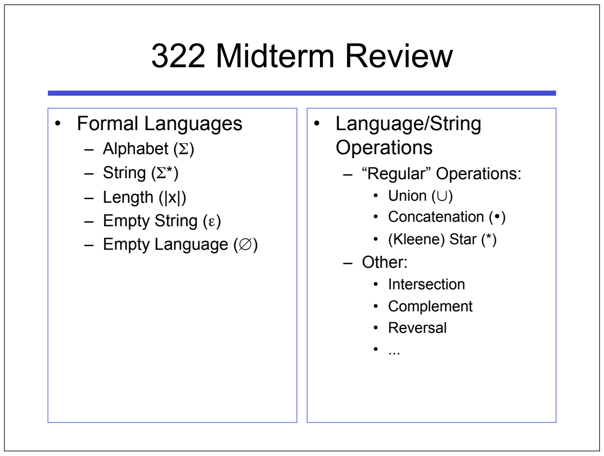
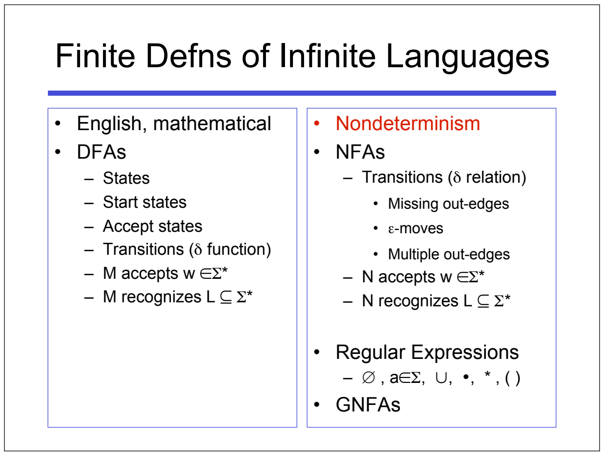


322 Midterm Review

- Formal Languages
  - Alphabet (Σ)
  - String (Σ^*)
  - Length ($|x|$)
  - Empty String (ε)
  - Empty Language ($\varnothing$)

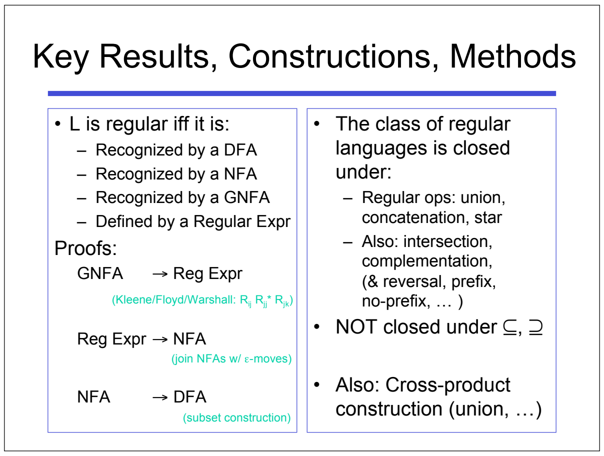
- Language/String Operations
  - "Regular" Operations:
    - Union ($\cup$)
    - Concatenation ($\bullet$)
    - (Kleene) Star ($*$)
  - Other:
    - Intersection
    - Complement
    - Reversal
    - ...



Finite Defns of Infinite Languages

- English, mathematical
- DFAs
  - States
  - Start states
  - Accept states
  - Transitions (δ function)
  - M accepts $w \in \Sigma^*$
  - M recognizes $L \subseteq \Sigma^*$

- Nondeterminism
- NFAs
  - Transitions (δ relation)
    - Missing out-edges
    - ε-moves
    - Multiple out-edges
  - N accepts $w \in \Sigma^*$
  - N recognizes $L \subseteq \Sigma^*$
- Regular Expressions
  - $\varnothing$, $a \in \Sigma$, $\cup$, $\bullet$, $*$, ( )
- GNFAs



Key Results, Constructions, Methods

- L is regular iff it is:
  - Recognized by a DFA
  - Recognized by a NFA
  - Recognized by a GNFA
  - Defined by a Regular Expr

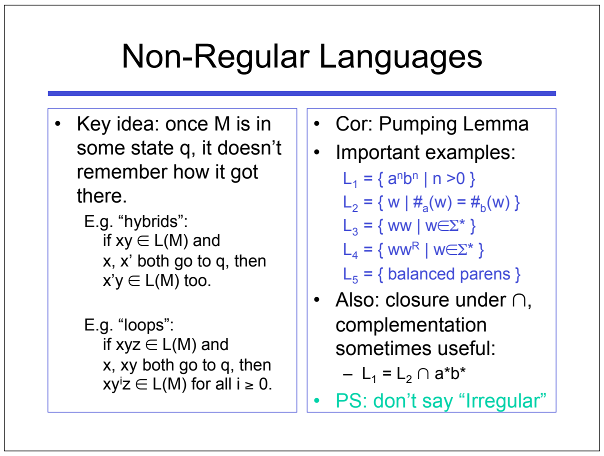
Proofs:

GNFA $\rightarrow$ Reg Expr
(Kleene/Floyd/Warshall: $R_{ij}\, R_{jj}^*\, R_{jk}$)

Reg Expr $\rightarrow$ NFA
(join NFAs w/ ε-moves)

NFA $\rightarrow$ DFA
(subset construction)

- The class of regular languages is closed under:
  - Regular ops: union, concatenation, star
  - Also: intersection, complementation, (& reversal, prefix, no-prefix, …)
- NOT closed under $\subseteq$, $\supseteq$
- Also: Cross-product construction (union, …)



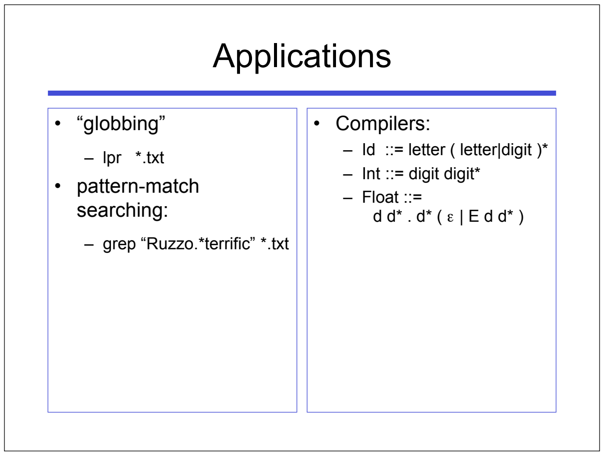
Non-Regular Languages

- Key idea: once M is in some state q, it doesn't remember how it got there.

  E.g. "hybrids":
  if $xy \in L(M)$ and
  x, x' both go to q, then
  $x'y \in L(M)$ too.

  E.g. "loops":
  if $xyz \in L(M)$ and
  x, xy both go to q, then
  $xy^iz \in L(M)$ for all $i \geq 0$.

- Cor: Pumping Lemma
- Important examples:
  - $L_1 = \{ a^nb^n \mid n > 0 \}$
  - $L_2 = \{ w \mid \#_a(w) = \#_b(w) \}$
  - $L_3 = \{ ww \mid w \in \Sigma^* \}$
  - $L_4 = \{ ww^R \mid w \in \Sigma^* \}$
  - $L_5 = \{$ balanced parens $\}$
- Also: closure under $\cap$, complementation sometimes useful:
  - $L_1 = L_2 \cap a^*b^*$
- PS: don't say "Irregular"



Applications

- “globbing”
  - lpr   *.txt
- pattern-match searching:
  - grep “Ruzzo.*terrific” *.txt

- Compilers:
  - Id ::= letter (letter|digit)*
  - Int ::= digit digit*
  - Float ::=
    d d* . d* (ε | E d d*)