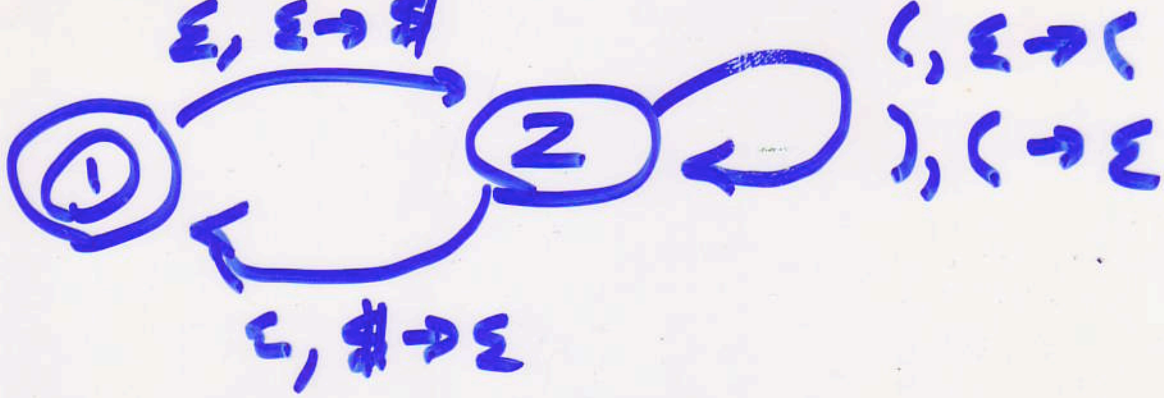


p	a	x	r	s	b	q
1	ε	$	2	2	ε	1
2	(	(	2	2	)	2

$A_{11} \to \varepsilon A_{22} \varepsilon$

$A_{22} \to (A_{22})$

$A_{11} \to \varepsilon$

$A_{22} \to \varepsilon$

$A_{11} \to A_{11} A_{11} \mid A_{12} A_{21}$

$A_{12} \to A_{11} A_{12} \mid A_{12} A_{22}$

$A_{21} \to A_{21} A_{11} \mid A_{22} A_{21}$

$A_{22} \to A_{21} A_{12} \mid A_{22} A_{22}$

NB: G can be simplified. Eg. remove A_{12}, A_{21} & all rules using them since, eg. there is no $x \in \Sigma^*$ s.t. $A_{21} \Rightarrow^* x$. This is just what we want in the construction, since there is no x s.t. $[2, \varepsilon, x] \vdash^* [1, \varepsilon, \varepsilon]$

<u>Claim</u> $\forall x \in \Sigma^* \; A_{pq} \overset{*}{\Rightarrow} x$

iff $[p, \varepsilon, x] \vdash^* [q, \varepsilon, \varepsilon]$

<u>Cor</u> $L(G) = L(M)$

Since $L(G) \underset{\uparrow \text{defn.}}{=} \{x \mid A_{init, final} \overset{*}{\Rightarrow} x\}$

$\underset{\uparrow \text{by claim}}{=} \{x \mid [init, \varepsilon, x] \vdash^* [final, \varepsilon, \varepsilon]\}$

$\underset{\uparrow \text{defn.}}{=} L(M)$

claim ($\Longleftarrow$) induction on deriv length

basis

$A_{pq} \overset{0}{\Rightarrow} x$: impossible; nothing to prove

$A_{pq} \overset{1}{\Rightarrow} x$: must be $x = \varepsilon$, $p = q$

$$[p, \varepsilon, \varepsilon] \vdash^* [q, \varepsilon, \varepsilon]$$

Ind

$\Rightarrow^{k+1}$ either

(i) $A_{pq} \to A_{pr} A_{rq} \Rightarrow^k x$

(ii) $\underbrace{A_{pq} \to a A_{rs} b}_{(*)} \Rightarrow^k x$

case (ii):

$$x = ayb \ \& \ A_{rs}$$

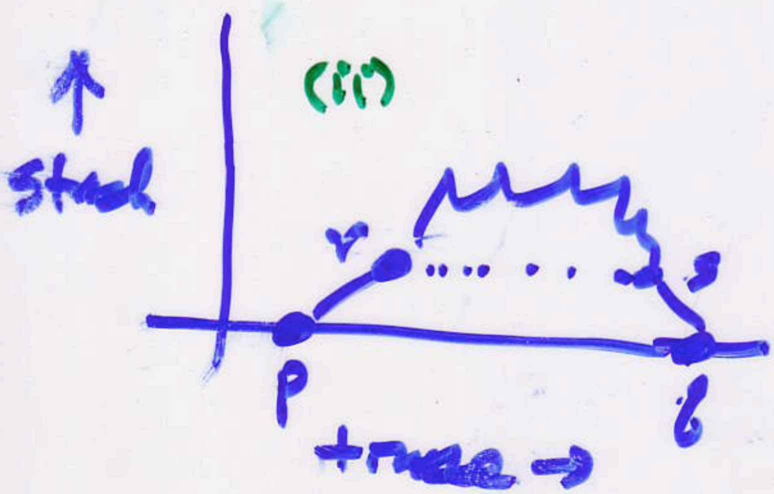
by ind $[r, \varepsilon, y] \vdash^* [s, \varepsilon, \varepsilon]$

$$[p, \varepsilon, ayb] \vdash [r, X, yb] \vdash^* [s, X, b]$$

since (*) in G

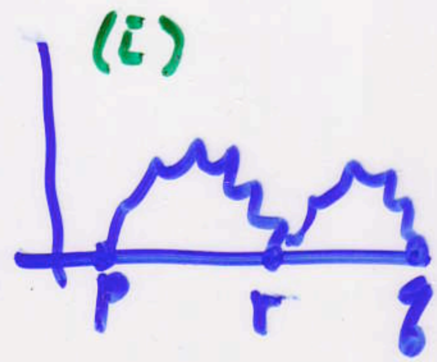
$$\vdash [q, \varepsilon, \varepsilon]$$

(ii)



or

(i)



$\Leftarrow$ direction of claim is similar, by induction on # of steps in $\vdash^*$

basis: 0 steps, use ε rule in G

ind: $k+1 > 0$ steps, then stack either is (case i) or is not (case ii) empty at some intermediate step.

In case i, I.H. & construction give $A_{pq} \to A_{pr} A_{rs}$ etc.

In case ii, $A_{pq} \to a A_{rs} b$ etc.

This construction & proof are just like the text's version, so more details there.