$3 + 4 * 5$

Syntax if no parens

$(3+4) * 5 + ((((0))))$

parens < 422 deep

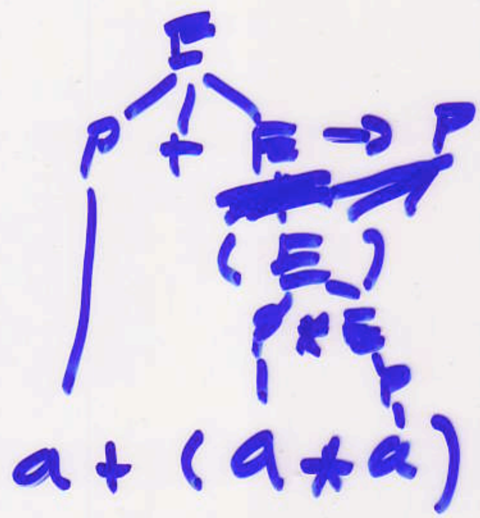
$E \to P$

$E \to P + E$

$E \to P * E$

$P \to a$

$P \to (E)$



$$
\begin{aligned}
\langle \text{SENTENCE} \rangle &\rightarrow \langle \text{NOUN-PHRASE} \rangle \langle \text{VERB-PHRASE} \rangle \\
\langle \text{NOUN-PHRASE} \rangle &\rightarrow \langle \text{CMPLX-NOUN} \rangle \mid \langle \text{CMPLX-NOUN} \rangle \langle \text{PREP-PHRASE} \rangle \\
\langle \text{VERB-PHRASE} \rangle &\rightarrow \langle \text{CMPLX-VERB} \rangle \mid \langle \text{CMPLX-VERB} \rangle \langle \text{PREP-PHRASE} \rangle \\
\langle \text{PREP-PHRASE} \rangle &\rightarrow \langle \text{PREP} \rangle \langle \text{CMPLX-NOUN} \rangle \\
\langle \text{CMPLX-NOUN} \rangle &\rightarrow \langle \text{ARTICLE} \rangle \langle \text{NOUN} \rangle \\
\langle \text{CMPLX-VERB} \rangle &\rightarrow \langle \text{VERB} \rangle \mid \langle \text{VERB} \rangle \langle \text{NOUN-PHRASE} \rangle \\
\langle \text{ARTICLE} \rangle &\rightarrow \texttt{a} \mid \texttt{the} \\
\langle \text{NOUN} \rangle &\rightarrow \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \\
\langle \text{VERB} \rangle &\rightarrow \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle \text{PREP} \rangle &\rightarrow \texttt{with}
\end{aligned}
$$

Grammar G_2 has 10 variables (the capitalized grammatical terms written inside brackets); 27 terminals (the standard English alphabet plus a space character); and 18 rules. Strings in $L(G_2)$ include

```
a boy sees
the boy sees a flower
a girl with a flower likes the boy
```

$$
\begin{aligned}
\langle \text{SENTENCE} \rangle &\Rightarrow \langle \text{NOUN-PHRASE} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \langle \text{CMPLX-NOUN} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \langle \text{ARTICLE} \rangle \langle \text{NOUN} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \texttt{a}\ \langle \text{NOUN} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \texttt{a boy}\ \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \texttt{a boy}\ \langle \text{CMPLX-VERB} \rangle \\
&\Rightarrow \texttt{a boy}\ \langle \text{VERB} \rangle \\
&\Rightarrow \texttt{a boy sees}
\end{aligned}
$$

A Context-free Grammar

$G = (V, \Sigma, R, S)$

V finite set (non-terminals)

Σ ... alphabet (terminals)

$V \cap \Sigma = \emptyset$

$S \in V$ Start non-terminal

R rules : <u>finite</u> subset

$\subseteq V \times (V \cup \Sigma)^*$

$V = \{E, P\}$

$\Sigma = \{a, +, *, (,)\}$

$S = E$

$R = \{ E \to P \cdots \}$

Convention

$\alpha, \beta, \gamma \ldots \in (V \cup \Sigma)^*$

$x, y, z \in \Sigma^*$

$a, b, c \in \Sigma$

$A, B, C \in V$

Defn

$\forall\ \alpha, \beta, \gamma \in (V \cup \Sigma)^*$

$\forall\ A \in V$

if $A \to \beta$ is $\in R$

Then

$$\alpha A \gamma \Rightarrow \alpha \beta \gamma$$

$\Rightarrow^*$

zero or more $\Rightarrow$

$$L(G) = \{ w \in \Sigma^* \mid S \Rightarrow^* w \}$$

L is a context-free Language if $\exists\ G$ s.t. $L = L(G)$

$S \to aSb \mid \varepsilon$

$B \to B\bar{B}a$

$L(G) = \{ a^n b^n \mid n \geq 0 \}$

$S \overset{k}{\Rightarrow} a^k S b^k \quad \forall k \geq 0$

by ind. on k

$a^k S b^k \Rightarrow a^k b^k$

$$L = \{ w \mid \#_a w = \#_b w \}$$

$S \to \varepsilon$
$S \to aSb\boxed{\underline{S}}$
$S \to bSa\boxed{\underline{S}}$
$S \to SS$

$S \to Sb$
$S \to Sa$
$S \to \varepsilon$

$\{a, b\}^*$

$a\,b\,b\,a$

① if $S \Rightarrow^K w \in \Sigma^*$

then $\#_a w = \#_b w \leq K$

basis $K = 1$ ✓
ind $K+1$ (I.H.: true for all derivs $\leq K$)

$S \Rightarrow^K w'$

$S \Rightarrow^{K_1} x$
$S \Rightarrow^{K_2} y$
$K_1 + K_2 = K$

[$S \Rightarrow aSb \Rightarrow^K w = aw'b$
or $S \Rightarrow bSa \Rightarrow^K w = bw'b$
[or $S \Rightarrow SS \Rightarrow^K w = xy$