Those who cannot remember the past are condemned to repeat it.

-- George Santayana (1905) Life of Reason

Given DFA M of

$xz \in L(M)$ &

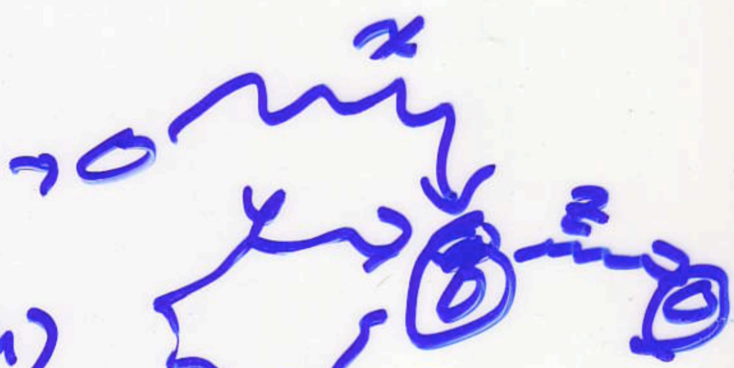
both x & x' take M to

same state (q)

Then $x'z \in L(M)$

if $x' = xy$

$\forall i \geq 0 \quad xy^i z \in L(M)$



Pumping Lemma

If L is reg. $\exists\, p > 0$ st

$\forall s \in L$ $|s| \geq p$ $\exists\, x, y, z \in \Sigma^*$

st $s = x \cdot y \cdot z$

$|y| > 0$

$|xy| \leq p$

$\forall i \geq 0$ $xy^i z \in L$

proof

let $L = L(M)$ for some DFA M

let p = # of states in M

let $s \in L$ with $|s| \geq p$

while reading 1st p letters of s, M visits $\geq p+1$ states

$|Q| = p$ so must be repeated states q

let x be prefix of s up to 1st arrival in state q; y part following that to 2nd arrival at q; z rest.

$L_1 = \{ 0^n 1^n \mid n \geq 0 \}$

To show: L_1 not regular; Assume it is.
Let p be the constant from P.L.

Consider $s = 0^p 1^p$

$\exists$ $x\,y\,z$ st $s = x \cdot y \cdot z$

$|y| \neq 0$

→ $|xy| \leq p$

$xy \in 0^*$

$\exists$ $i, j > 0$ st $x = 0^i, y = 0^j$

$z = 0^{p-i-j} 1^p$

$\forall i$ ~~xy^iz~~ ...

$xz \in L(M)$ $xz = 0^{p-j} 1^p \notin L_1$

because
$p - j \neq p$
$(j > 0)$

xy^2z ~~...~~
$\in L(M)$
$\notin L_1$

$xy^2z = 0^{p+j} 1^p$

choice of s not unique

$s = 0^{\lceil p/2 \rceil} 1^{\lceil p/2 \rceil}$

$\exists\ xyz \ldots\ s = xyz$

$|y| \neq 0$

$|xy| \leq p$

Case 1

$xy \in 0^*$ ... as before

Case 2

$y \in 1^*$ $\quad xy^2z \notin L_1$

Case 3 $y = 0^k 1^l$ some $k, l \geq 1$

$xz \rightarrow ?$

$xy^2z = 0^{\lceil p/2 \rceil} 1^l 0^k 1^{\lceil p/2 \rceil}$

$x = 0^{\lceil p/2 \rceil - k}$

$\notin L_1$ because

$\underbrace{\overbrace{0 \ldots}^{p/2 - k}}_{x} \; \underbrace{0^k 1^l}_{y} \; \underbrace{0^k 1^l}_{y} \; \underbrace{1^{p/2 - l}}_{z}$ 0 following a 1.

$L_2 = \{ a^q \mid q \text{ is prime} \}$

get p

$q =$ 1st prime $> p$

$S = a^q \in L_2$

$\exists \; x y z \; \ldots$

$|xy| \le p$

$i \ge 1 \quad x y^i z = a^{q + (i-1)|y|}$

$i = q+1 \rightarrow a^{q + (q+1-1)|y|}$

$= a^{q(|y|+1)} \notin L_2$

not prime