

What's not regular?

Not Reg. $\left[\{ 0^n 1^n \mid n \geq 0 \} \right]$

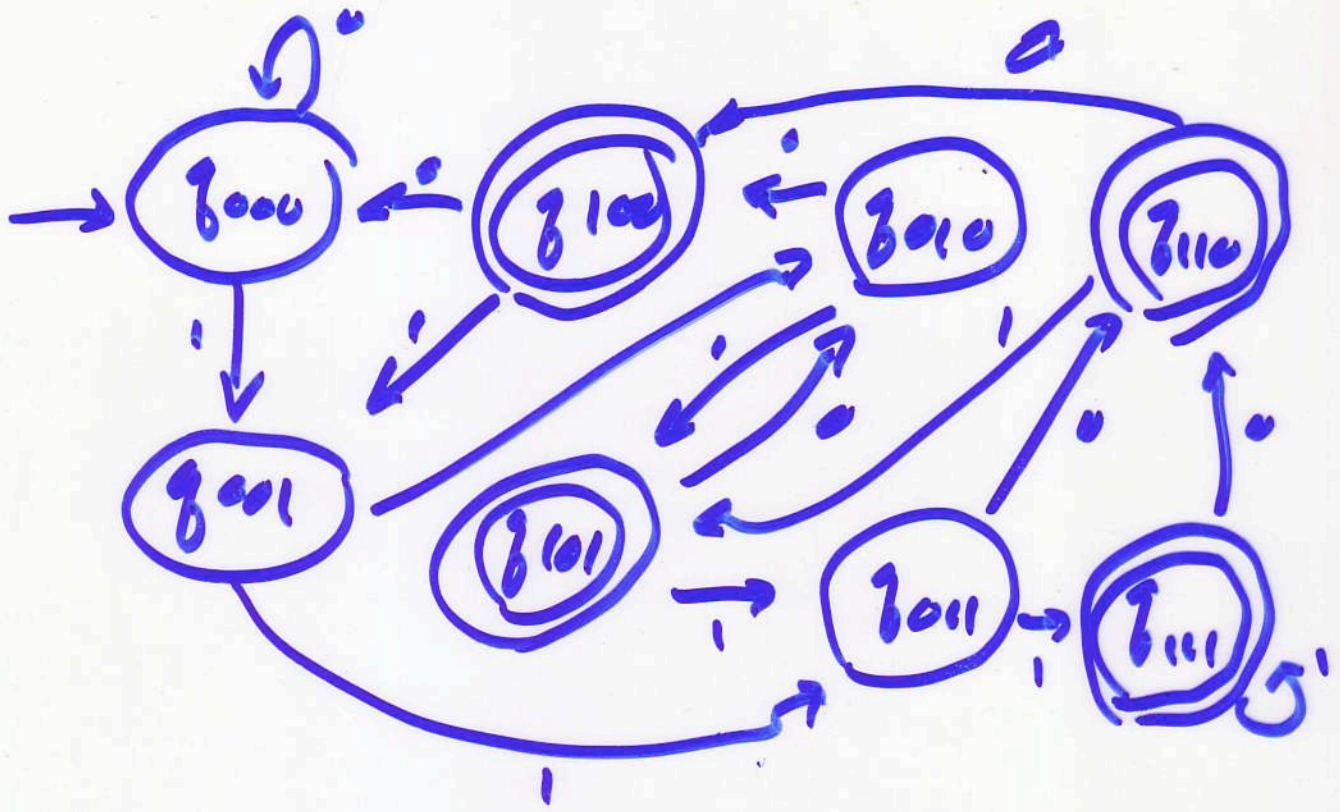
Reg. $\left[\{ x \mid \#_0(x) = \#_1(x) \} \right]$

is Reg. $\left[\{ x \mid \#_{01}(x) = \#_{10}(x) \} \right]$

0101

 1

$\{ x \mid 3^{\text{rd}} \text{ letter from right is } 1 \}$



Suppose L is regularized by M with $k < 8$ states
 Consider 8 inputs

if 00x0
 01x0
 ↑
 RR 00
 RR 00
 ↑

000
 001
 010
 011
 100
 101
 110
 111

By P.h.p.
 2 inputs →
 Same state
 ∴ if 0xx
 2 1xx
 go to same then
 error $q \in F$ not.

$$\{0^n 1^n \mid n \geq 0\}$$

Take any M , DFA.

Suppose M has p states.

Let $q_i =$ state M is in
after reading 0^i
 $0 \leq i \leq p$

$p+1$ values of i , so for some
 $j < k$
 $0 \leq j, k \leq p$ $q_j = q_k$ $j \neq k$

If M accepts $0^j 1^j$

then it also accepts

$0^k 1^j$.

$\therefore L(M) \neq \{0^n 1^n \mid n \geq 0\}$

$0^{j+(k-j)} 1^j$

$$\{ 0^n | n \geq 0 \} \quad \{ x | \#0x = \#1x \}$$

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$p+1$ values of i , so for some
 $j < k$
 $0 \leq j, k \leq p \quad q_j = q_k \quad j \neq k$

If M accepts 0^j

then it also accepts

0^k .

$\therefore L(M) \neq \{ 0^n | \#0^n = \#1^n \}$
 $0^{j+(k-j)} = 0^k$

$$L = \{ ww^R \mid w \in \Sigma^* \}$$

aba abaa

Suppose
~~is~~ M w/ p states accepts L .

let ~~let~~ $w_1 \dots w_{p+1} ba$
 $p+1$

let q_i be state M is in
 after reading a^i $0 \leq i \leq p$

$bba^i \dots$ $q_j = q_k$

$a^j b b a^i$

$a^k b b a^i$ etc.

$w_1 \dots w_{p+1}$ all of same length
 all different

$w_j w_j^R \in L$

$w_k w_j^R \notin L$

Pumping Lemma

If L is reg. $\exists p > 0$ st
 $\forall w \in L$ $|w| \geq p \exists x, y, z$
st $w = x \cdot y \cdot z$

$$|y| \geq 1$$

$$|xy| \leq p$$

$$\forall i \geq 0 \quad xy^i z \in L$$