CSE 322
Intro to Formal Models in CS
Simulation of NFAs by DFAs: Notes on the Proof of Theorem 1.39

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8 Oct 07

The text's assertion that the construction given in the proof of Theorem 1.39 (1st ed: 1.19) is "obviously correct" seems perhaps a little breezy. Here is an outline of a somewhat more formal correctness proof. I will only handle the case where the NFA has *no* ϵ-transitions. Notation is as in the book.

For any $x \in \Sigma^*$, define

$$\begin{aligned} Q_{N,x} &= \{r \in Q \mid N \text{ could be in state } r \text{ after reading } x\}, \text{ and} \\ Q_{M,x} &= \text{ the state } R \in Q' \text{ that } M \text{ would be in after reading } x. \end{aligned}$$

The key idea in the proof is that these two sets are identical, i.e. that the single state of the DFA faithfully reflects the complete range of possible states of the NFA. The proof is by induction on $|x|$.

BASIS: ($|x| = 0$.) Obviously $x = \epsilon$. Then

$$Q_{N,\epsilon} = \{q_0\} = q_0' = Q_{M,\epsilon}.$$

The first and third equalities follow from the definitions of "moves" for NFAs and DFAs, respectively, and the middle equality follows from the construction of M.

INDUCTION: ($|x| = n > 0$.) Suppose $Q_{N,y} = Q_{M,y}$ for all strings $y \in \Sigma^*$ with $|y| < n$, and let $x \in \Sigma^*$ be an arbitrary string with $|x| = n > 0$. Since x is not empty, there must be some $y \in \Sigma^*$ and some $a \in \Sigma$ such that $x = ya$. For any $r \in Q$,

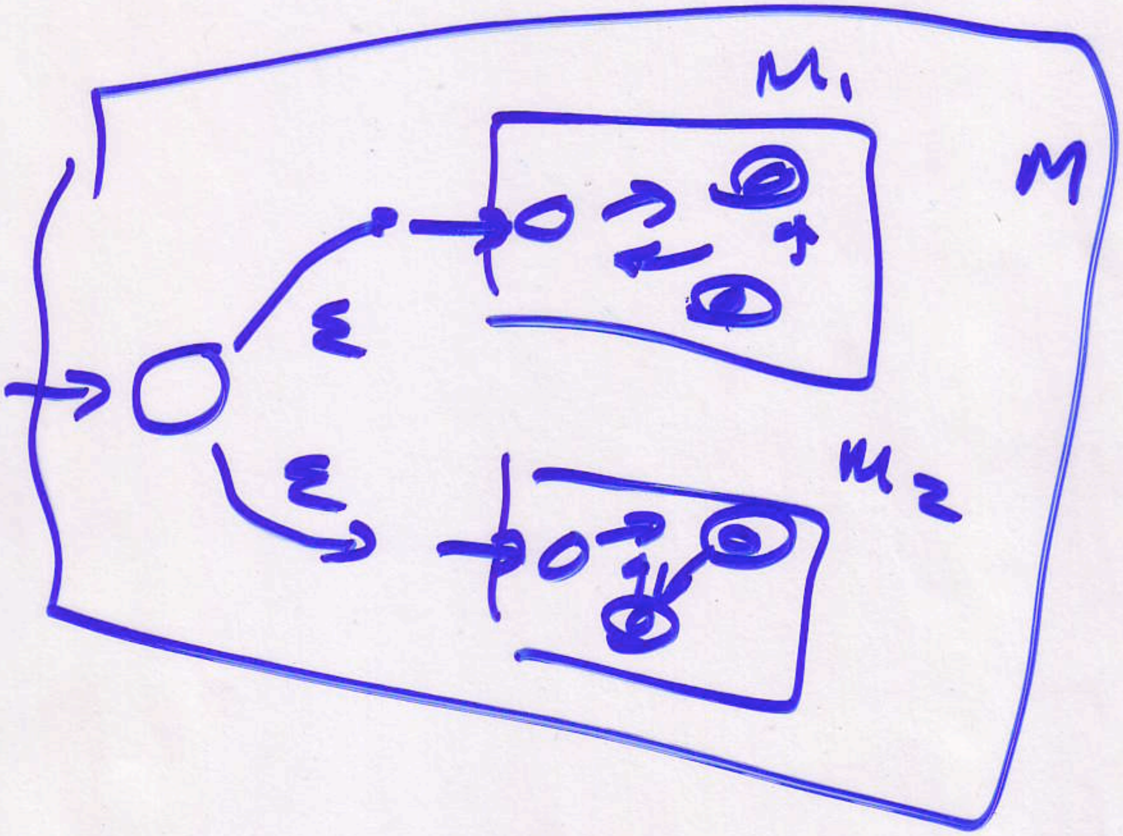
$$\begin{aligned}
& N \text{ could be in state } r \text{ after reading } x = ya && (1)\\
\Leftrightarrow\ & \text{there is some } r' \in Q \text{ such that } N \text{ is could be in } r' \text{ after reading } y \text{ and } r \in \delta(r', a) && (2)\\
\Leftrightarrow\ & r \in \bigcup_{r' \in Q_{N,y}} \delta(r', a) && (3)\\
\Leftrightarrow\ & r \in \delta'(Q_{N,y}, a) && (4)\\
\Leftrightarrow\ & r \in \delta'(Q_{M,y}, a) && (5)\\
\Leftrightarrow\ & r \in Q_{M,x} && (6)
\end{aligned}$$

The equivalence of (1) and (2) follows from the definition of "moves" for NFAs: the last step must be a move from some stae reached after reading y. The equivalence of (2) and (3) is just set theory. The equivalence of (3) and (4) follows from the definition of δ'. The equivalence of (4) and (5) follows from the induction hypothesis. The equivalence of (5) and (6) follows from the definition of ”moves“ for DFAs.

Given the equivalence established above, it's easy to see that $L(N) = L(M)$, since N accepts x if and only if it can reach a final state after reading x, which will be true if and only if $Q_{N,x}$ contains a final state, which happens if and only if $Q_{M,x} \in F'$.

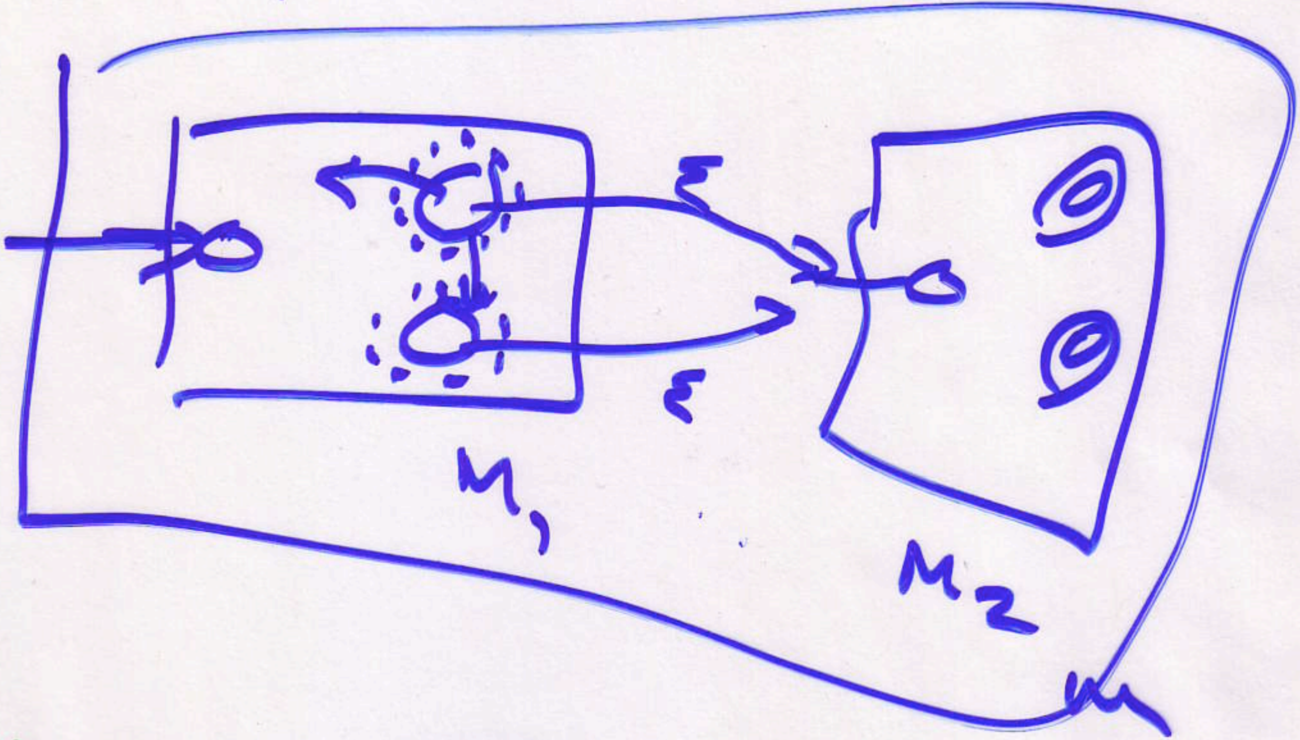
if M_1 & M_2 are F.A. recognizing $L_i = L(M_i)$

Then the following recognize $L_1 \cup L_2$



2 NFA's M_1 M_2 $L_i := L(M_i)$

$L_1 \cdot L_2$



I. Suppose $x \in L_1$, $y \in L_2$

Then M_1 reading x can reach a final state, say q. By construction, $q_{20} \in \delta(q, \varepsilon)$ (where q_{20} = init of M_2)

And from $q_{0,2}$ reading y, M_2 reaches a final state. ∴ M reading xy can reach a final state so xy accepted by M

∴ $\boxed{L_1 \cdot L_2 \subseteq L(M)}$