Closure under $\cup$

even parity $\subseteq \{0,1\}^*$

or 3rd from right is a $\subseteq \{a,b\}^*$

... - - - - - - 1 $\subseteq \{0,1\}^*$

$M_i = (Q_i, \Sigma, \delta_i, q_{0i}, F_i)$

$M = (Q_1 \times Q_2, \Sigma, \delta, (q_{01}, q_{02}), F)$

$\forall\ q_1 \in Q_1, q_2 \in Q_2\ \ a \in \Sigma$

$$\delta((q_1,q_2),\underbrace{a}_{\Sigma}) = (\delta_1(q_1,a), \delta_2(q_2,a))$$

$F = F_1 \times Q_2 \cup Q_1 \times F_2$ — union

$F' = F_1 \times F_2$ — intersection

(*) To show by induction on length of w $\forall q_1 \in Q_1, q_2 \in Q_2, \forall w \in \Sigma^*$

M is in state $(q_1, q_2)$ after reading w $\Leftrightarrow$ $M_1$ is in state $q_1$ after reading w and $M_2$ is in state $q_2$ after reading w

<u>Cor</u>

M accepts $L(M_1) \cup L(M_2)$

* Note that there is a key difference between statement (*) and the definition of M: the later, via defn of $\delta$, says something about the finite set of ~~con~~ letters in $\Sigma$; (*) talks about the infinite set of strings in $\Sigma^*$.

$X, Y \subseteq \Sigma^*$

$X \cdot Y = \{x \cdot y \mid x \in X \,\&\, y \in Y\}$

Examples

$L_{oddparity} \cdot L_{oddparity} = L_{even} - \{0\}^*$

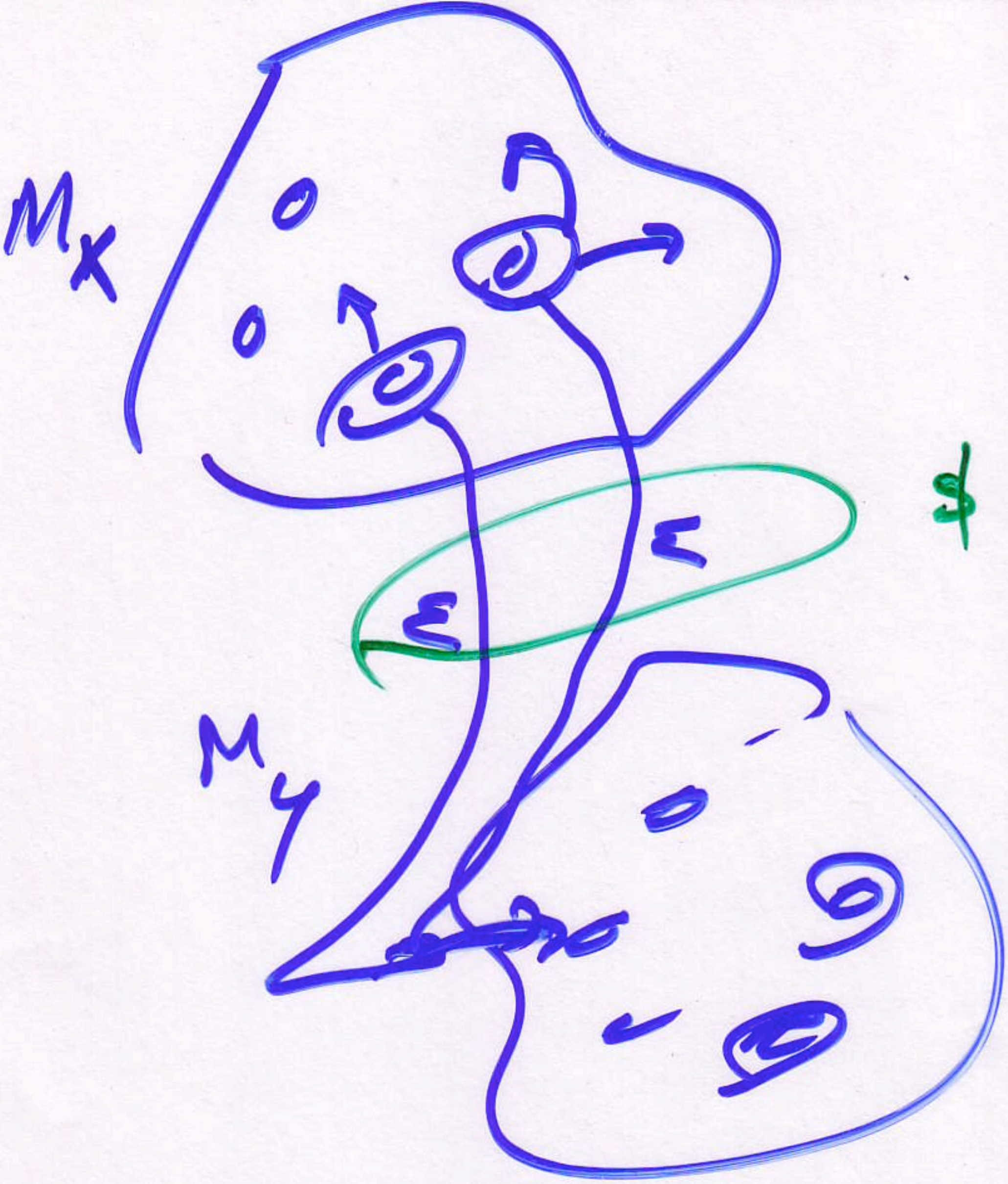
$L_{oddparity} \cdot L_{even} = L_{odd}$

$\underset{\text{infinite}}{A} \cdot \underset{\text{finite}}{B} \stackrel{?}{=} \underset{\text{Finite}}{C}$ ] possible?

$\Sigma^* \cdot \phi = \phi$ ] yes

$X \cdot Y \stackrel{?}{=} Y \cdot X$ ] always true?

$\{0\} \cdot \{1\} \neq \{1\} \cdot \{0\}$ ] no




$X, Y \subseteq \{0,1\}^*$ , $\$ \notin \{0,1\}$

$X \cdot \{\$\} \cdot Y$ ← "marked concatenation"