DFA Minimization

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Announcements

- Turn in H/W #4
- Handouts
  - Midterm exam topics list
  - Sample Midterm 1
  - Sample Midterm 2
  - Feedback Form
  - Solutions to H/W # 3
    - If you did not pick up one last time
- No homework this week

Puzzle

- Use Myhill-Nerode theorem to prove that the following language is not regular:
- $L=\{ 0^n \mid n \text{ is prime} \}$

The equivalence relation $\equiv_A$

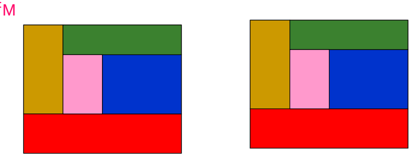
- Let A be a language
- Given $\equiv_A$, we know how to build minimzed DFA for A
- Recall
  - For strings x and y, $x \equiv_A y$ iff
  - For all strings z, either both xz and yz are in A or both are not in A

The equivalence relation $\equiv_M$

- Given DFA M
- Recall
  - String x and y, $x \equiv_M y$ iff
  - x and y end up in the same state in M
- Each equivalence class corresponds to a state
- Assume all states in M are reachable from the start state

Relationship between $\equiv_A$ and $\equiv_M$

- Let M be such that A = L(M)
- If $x \equiv_M y$ then $x \equiv_A y$
- An eqv class in $\equiv_A$ is the union of eqv classes in $\equiv_M$



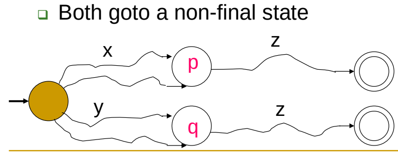
$\equiv_M$ $\equiv_A$

Basic idea

- In the minimized DFA, every state corresponds to a eqv. class in $\equiv_A$
- But we only know $\equiv_M$
- Group states in M to get states corresponding to $\equiv_A$

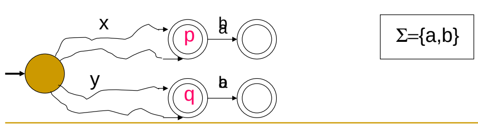
When should we group states ?

- Given two states p and q, when should we group them ?
- When $x \equiv_A y$
- If for all strings z, either
  - Both p and q goto a final state, or
  - Both goto a non-final state



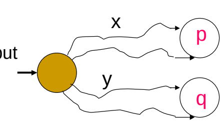
In other words…

- For all strings z, of length 0
  - Both p and q are final states or both are not
- For all strings z, of length 1
  - Both p and q are final states or both are not
- For all strings z, of length 2 ….



Putting it together…

- Group p and q together, if
  - For all $i \geq 0$
    - For all strings z of length i, both p and q are in either final states or not
- Do not group p and q, if
  - There exist an $i \geq 0$
    - Exists a string z of length i that takes p to a final state but not q (or vice versa)



Stating it more formally

- $p \equiv_i q$ if
  - For all strings z of length at most i, either both p and q reach final state or neither does
- Thus, group p and q if
  - $p \equiv_i q$ for all $i \geq 0$
- Do not group p and q if
  - $p \not\equiv_i q$ for some $i \geq 0$

Questions ?

The procedure

- We will decide in a top down manner
- First group all states together
- Separate the states that are not equivalent under $\equiv_0$
- Separate the states that are not equivalent under $\equiv_1$
- And so on…

And so on till when ?

What we need…

- The process needs to terminate
- At termination, all grouped states belong to the same eqv class in $\equiv_A$
- Any two state in different groups must be in different eqv classes in $\equiv_A$