CSE322: Formal Models in Computer Science Spring 2006

PROBLEM SET 6

Due Friday, May 19, 2006, in class

Reading Assignment: Read Sections 2.2 and 2.3 of Sipser's text.

Instructions: The basic instructions are the same as in Problem Set 1.

There are **FIVE** questions in this assignment. Again, it never hurts to start early. Also **do not forget** to mention the names of your collaborators in your homework.

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1. ($4 \times 10 = 40$ points) Draw PDAs for the following languages and briefly justify your construction .
   (a) (*) The set of strings over $\{0, 1\}$ that have thrice as many 1's as 0's.
   (b) (*) $\{w \in \{0, 1\}^* \mid w^R = w$, that is, w is a palindrome $\}$.
   (c) The complement of the language
   $A = \{w \in \{a, b, c\}^* \mid w$ has equal number of a's, b's and c's $\}$.
   We will later see in the course that A is not context-free. Thus, this is an example that shows that context-free languages are not closed under intersection.
   (d) $\{w \in \{(,)\} \mid w$ is a string of balanced parens except for *exactly* one extra ($\}$.
   For example (()((())) is in the language while ((() and (()(())) are not. (In one of the puzzles you were asked to design a grammar for this language).
   (*Hint* : It might be easier to start with the PDA that generates strings with balanced parens and try to modify that PDA to generate strings in the above language.)

2. (10 points) Prove that the intersection of a context free language and a regular language is always context free.

   (*Hint* : Recall a language is context free if it is is accepted by a PDA.)

3. (10 points) Is the following statement true:

   If M is a PDA and there exists a natural number k such that for all $w \in L(M)$, the size of the stack is at most k in any computation of M on w, then $L(M)$ is not regular.

   Justify your assertion.

4. (*) (10 points) Carry out the general transformation to convert a CFG to a PDA for the following grammar that generates balanced parens:

   $$S \to (S) \mid SS \mid \epsilon.$$

   (Do *not* use the shorthand for transition on strings that was used in the proof in class.)

5. (**Bonus**) (10 points) Prove that the following language is context-free

   $$\{x\#y \mid x, y \in \{0, 1\}^* \text{ and } x \neq y\}.$$

   (You do not have to formally prove that your grammar (or PDA) generates (accepts) the above language but you should give an argument why your construction works.)