CSE 322 Spring 2005
Assignment #2

Due: Friday, April 15, 2005

Reading assignment: Finish reading Chapter 1 of Sipser's book sections 1.1-1.3.

Problems:

1. Given two strings x and y of exactly the same length we can create a new string $shuffle(x, y)$ that consists of the characters of x and y alternating one after another starting with the first character of x. That is if $x = a_1 \dots a_k$ and $y = b_1 \dots b_k$ then $shuffle(x, y) = a_1 b_1 a_2 b_2 \dots a_k b_k$. For languages K and L define

$$SHUFFLE(K, L) = \{shuffle(x, y) \ : \ x \in K, \ y \in L, \ |x| = |y|\}.$$

   Given DFAs that recognize K and L give a brief intuitive description and then a formal description of how to build a DFA that recognizes $SHUFFLE(K, L)$.

2. Sipser's book page 88, Problem 1.27. Document the states of your DFA.

3. Sipser's book page 89, Problem 1.30. Explain what your states will be and describe how the transition function will be defined depending on n.

4. Draw NFAs with at most 8 states that recognize each of the following languages. Explain why each of your NFAs is correct. (Full state-by-state documentation is not required.)

   (a) The set of all binary strings containing 0110 or 101.

   (b) The set of all binary strings with a 1 in the 4th from last position.

   (c) The set of all binary strings other than 010 or 101.

5. For a language L define the reverse of L, $L^R = \{x^R \mid x \in L\}$. Give a construction that will take a DFA M that recognizes L and convert it to an NFA M' that recognizes L^R. Give a formal definition of M' based on M and briefly argue why your construction is correct.

6. (Bonus) Sipser's book page 89, Problem 1.31.

7. (Bonus due April 22) Show that if L is recognized by a finite automaton there is a finite automaton that recognizes the set of first halves of strings in L,

$$L_{\frac{1}{2}} = \{x \ : \ xy \in L \text{ for some } y \text{ with } |x| = |y|\}.$$