CSE 322 Spring 2004

<u>Homework Assignment # 1</u>

Due Date: Friday, April 9 (at the *beginning* of class)

Note: N = set of natural numbers = {1, 2, 3, …}, Z = set of integers = {…, -1, 0, 1, …}

1. (15 points) Write formal descriptions of the following sets:
   Examples: Set containing 1, 10, 100 = {1, 10, 100}
   Set containing all even integers = $\{n \mid n = 2m \text{ for some } m \in Z\}$
   a. The set containing all integers that are greater than –5 and less than 0
   b. The set containing all natural numbers that are greater than –5 and less than 0
   c. The set containing all integers that are odd and divisible by 3
   d. The set containing all strings (over $\Sigma = \{0,1\}$) that do not contain 111 as a substring
   e. The set containing all strings (over $\Sigma = \{0,1\}$) of odd length whose middle symbol is 0

2. (20 points) Let $A = \{n \mid n \text{ is a prime number and } 10 < n < 20\}$ and let $B = \{n \mid n = 2m-1 \text{ for some } m \in N \text{ and } 5 < m < 10\}$.
   a. Which of the following 4 statements is/are true:
      i. $A \subseteq B$
      ii. $B \subseteq A$
      iii. $A \cap B \neq \varnothing$
      iv. $A \cup B = \{n \mid n = 2m+1 \text{ for some } m \in N \text{ and } 5 \leq m < 10\}$
      v. $(B - A) = \{15,19\}$ (Recall the definition of "–" for sets discussed in class in the proof example for set equality)
   b. What is the complement of $A \cup B$? (Write a formal description in the form: {n | … }. Take complement with respect to N)
   c. What is $A \times B$? (List all the elements)
   d. Is $A \times B = B \times A$? Why/Why not?
   e. What is the power set of $A \cap B$ (i.e. $2^{A \cap B}$)? (List all the elements)

3. (15 points) Let A, B, and C be any three sets. Prove or disprove:
   $A - (B \cap C) = (A - B) \cap (A - C)$

4. (20 points) Prove that for any two integers x and y, xy is odd if and only if both x and y are odd.

5. (30 points) Prove the following:
   a. For any two strings x and y in Σ^*, $(xy)^R = y^R x^R$ (Hint: Use induction on the length of y).
   b. For any string x in Σ^* and any $k \geq 0$, $(x^k)^R = (x^R)^k$ (Hint: Use induction on k).