CSE 322
Winter Quarter 2003
Assignment 9
Due Friday, March 14, 2003

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Design a Turing machine the copies a string. The machine starts with a string $x \in \{0,1\}^*$ on the tape with the head on the first symbol of x. When the Turing machine halts the string xcx is written on the tape with the head on the first symbol of the output. You may use a state diagram as your design, but explain what the various states mean.

2. (10 points) Given a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ that halts on every input, design a Turing machine M' that also halts on every input and $L(M') = \Sigma^* - L(M)$. Recall that final states q have the property that $\delta(q, a)$ is undefined for all $a \in \Gamma$. Thus, it does not suffice to make all the non-final states final.

3. (10 points) Let $G = (V, \Sigma, P, S)$ be a context-free grammar. Using a closure algorithm we know how to compute the productive non-terminals, those non-terminals that generate a terminal string. Assume we have an algorithm for computing the *positive productive non-terminals*, that is, all non-terminals A such that $A \Rightarrow^* x$ for some $x \in \Sigma^+$. For $A \in V$, define the relation $R(A, B)$ if $A \Rightarrow^* xBy$ for some $xy \in \Sigma^*$ and $R^+(A, B)$ if $A \Rightarrow^* xBy$ for some $xy \in \Sigma^+$.

   (a) Design a closure algorithm to compute $R(A, B)$ for all $A, B \in V$. You may use the algorithm for computing $Prod$, the productive non-terminals of G as a subroutine.

   (b) Design a closure algorithm to compute $R^+(A, B)$ for all $A, B \in V$. You may use the algorithm for computing the relation R, the algorithm for computing $Prod$, and the algorithm for computing $Prod^+$ (the set of positive productive non-terminals) as subroutines in your algorithm.

   (c) Using R, R^+, and $Prod$ as subroutines design an algorithm for determining if the language generated by G is infinite. Note that $L(G)$ is infinite if and only if there are terminal strings u, v, x, y, z and a non-terminal A such that $S \Rightarrow^* uAz$, $A \Rightarrow^* vAy$ with $vy \neq \epsilon$, and $A \Rightarrow^* x$.