CSE 322
Winter Quarter 2003
Assignment 7
Due Friday, February 28, 2003

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) In this problem we examine deterministic PDAs. A PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is a *deterministic* PDA (DPDA) if it is not possible for M to make two moves from the same ID. That is, M has the properties: (i) $|\delta(q, a, X)| \leq 1$ for $q \in Q$, $a \in \Sigma \cup \{\epsilon\}$, $X \in \Gamma$ and (ii) not both $\delta(q, a, X) \neq \phi$ and $\delta(q, \epsilon, X) \neq \phi$ for $q \in Q$, $a \in \Sigma$, $X \in \Gamma$. The second condition assures us that M cannot make both an ϵ-transition and a regular transition from the same ID. Let \$ be a new terminal symbol. We say that L is *deterministic context-free* if $L\$$ accepted by a DPDA. The symbol \$ is needed so the DPDA can detect the end of the input. Design a DPDA that accepts the language $L\$$ where $L = \{0^n 1^m 2^{2n+m} : n, m \geq 0\}$.

2. (10 points) In this problem we look at the emptiness problem for context-free grammars. Let $G = (V, \Sigma, R, S)$. A nonterminal A is *productive* if $A \Rightarrow_G^* w$ for some $w \in \Sigma^*$. That is, some terminal string can be generated from A.

   (a) Design a closure algorithm for finding all the productive nonterminals in a grammar G.

   (b) Use the algorithm in part (a) as part of an algorithm for deciding if the language generated by a context-free grammar is empty.

   (c) Use the algorithm in part (a) to construct a context-free grammar G' such that $L(G) = L(G')$ and for all α, if $S \Rightarrow_{G'}^* \alpha$ then $\alpha \Rightarrow_{G'}^* w$ for some $w \in \Sigma^*$. That is, G' and G generate the same language and in G' every partial derivation can be eventually completed into the derivation of some terminal string

3. (10 points) In this problem we show that the context-free languages are closed under intersection with a regular language. Let $M_1 = (Q_1, \Sigma, \Gamma, \delta_1, q_1, Z_0, F_1)$ be a PDA and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA. Use a cross product construction to build a PDA M such that $L(M) = L(M_1) \cap L(M_2)$. One important issue to deal with in your construction is that M_1 may have ϵ-transitions while M_2 does not. This shows that the context-free languages are closed under intersection with regular languages.