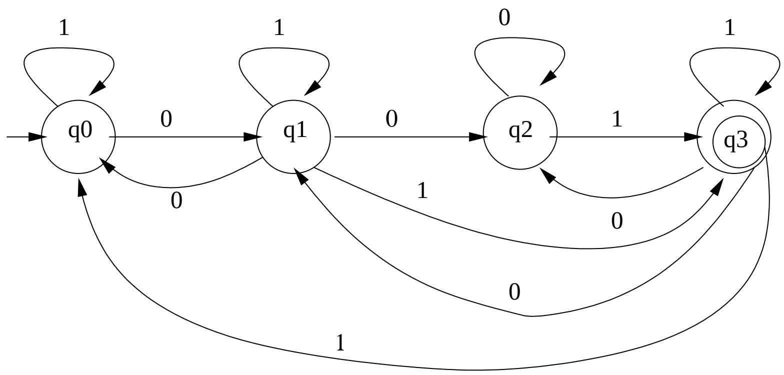
CSE 322
Winter Quarter 2003
Assignment 3
Due Friday, January 24, 2003

All solutions should be neatly written or type set. All major steps in proofs must be justified.

1. (10 points) For this problem you will practice converting a NFA to a DFA. Convert the following NFA to a DFA. Show only the reachable states of the DFA. The transition function should be given in a table.



2. (10 points) For this problem you will have practice in showing that regular languages are closed under more operations using finite automata constructions. We define the *interleaving* of two languages A and B over Σ by

$$A \| B = \{x_1 y_1 \cdots x_n y_n : x_i, y_i \in \Sigma^*, x_1 x_2 \cdots x_n \in A, \text{ and } y_1 y_2 \cdots y_n \in B\}.$$

For example if $A = \{a, ab\}$ and $B = \{01\}$ then $A \| B = \{a01, 0a1, 01a, ab01, a0b1, a01b, 0ab1, 0a1b, 01ab\}$. Show that if A and B are regular then so is $A \| B$. Start with DFA's M_1 and M_2 that accept A and B, respectively. Then construct an NFA that accepts $A \| B$. A cross product type construction will be useful.

3. (10 points) This problem is designed to help you think more abstractly about algorithms that will be useful in the analysis of finite automata. Given a directed graph $G = (V, E)$ and a vertex $v \in V$ define the set of vertices reachable from v as follows:

$$R(v) = \{v_k \ : \ v_0 = v \text{ and } (v_0, v_1, \ldots, v_k) \text{ is a path in } G \text{ for some } v_0, \ldots, v_{k-1}\}.$$

Notice that $v \in R(v)$ by letting $k = 0$ in the definition. Consider the following "closure algorithm" for computing $R(v)$.

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X = {v};
repeat
  X' = X;
  X = X' union {y : (x,y) in E and x in X'};
until X = X'
R(v) = X
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(a) Consider the graph $G = (V, E)$ with $V = \{1, 2, 3, 4\}$ and $E = \{(1, 2), (1, 3), (2, 4), (3, 2), (4, 3)\}$. Run the algorithm for $R(1)$, $R(2)$, $R(3)$ and $R(4)$ showing the result after each iteration of the repeat loop.

(b) If G has n vertices then what is the maximum number of times the repeat loop can be executed?

(c) Modify the algorithm for $R(v)$ to compute $R^+(v)$ which is the set of vertices reachable from v with a path of length at least 1, that is,

$$R^+(v) = \{v_k \ : \ v_0 = v \text{ and } (v_0, v_1, \ldots, v_k) \text{ is a path in } G \text{ for some } v_0, \ldots, v_{k-1} \text{ and } k > 0\}.$$

Note that if $v \in R^+(v)$ then there is a cycle in G which includes v. Recall that a cycle is a path whose first vertex matches last vertex.

(d) Notice that $v \in R(u)$ if and only if there is a path from u to v. Furthermore $v \in R^+(u)$ if and only if there is a path from u to v of positive length. Use R and R^+ to compute for a pair of vertices u and v whether or not there are infinitely many paths from u to v. For this problem you should give an algorithm, that calls R and R^+ as subroutines, for determining for a given u and v if there are infinitely many paths from u to v. You should also explain why your algorithm is correct.