CSE 322
Winter Quarter 2003
Assignment 2
Due Friday, January 17

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Design deterministic finite automata using a state transition diagram for each of the following languages.

   (a) $\{x \in \{0,1\}^* \ : \ 010$ is a substring of $x\}$.

   (b) $\{x \in \{0,1\}^* \ : \ 111$ is not a substring of $x\}$.

   (c) $\{x \in \{0,1\}^* \ : \ x$ contains exactly 5 $0's\}$.

   (d) $\{x \in \{0,1\}^* \ : \ x$ has an even number of $0's$ or an odd number of $1's\}$.

2. (10 points) Consider the languages

$$L_k = \{x \in \{0,1\}^* \ : \ x \text{ contains exactly } k \ 0's\}$$

   for $k \geq 0$.

   (a) Formally define a deterministic finite automaton M_k with exactly $k+2$ states that accepts L_k.

   (b) Prove by contradiction that every deterministic finite automaton that accepts L_k has at least $k+2$ states. The ideas from problem 2 of assignment 1 are useful.

3. (10 points) A finite state transducer $M = (Q, \Sigma, \Gamma, \delta, q_0)$ is defined by: Q is a finite set of states, Σ and Γ are alphabets and $\delta \ : \ Q \times \Sigma \to Q \times \Gamma^*$. That is, $\delta(q, \sigma) = (p, y)$ means that on input q processing $\sigma \in \Sigma$, M goes to state p and outputs the string y. Let $w = w_1 w_2 \cdots w_n$ where $w_i \in \Sigma$. We write $q \stackrel{w,y}{\to} p$ if there are states $r_0, \ldots, r_n$ and $y_1, \ldots, y_n \in \Gamma^*$ such that:

$$\begin{aligned} y &= y_1 y_2 \cdots y_n, \\ r_0 &= q, \\ r_n &= p, \\ (r_i, y_i) &= \delta(r_{i-1}, w_i), \ 1 \leq i \leq n. \end{aligned}$$

   For $x \in \Sigma^*$, define $f_M(x) = y$ if $q_0 \stackrel{x,y}{\to} p$ for some $p \in Q$. The string $f_M(x)$ is called the output of M on input x.

Design a finite state transducer that outputs the quotient in binary of a number written in binary divided by 3. For example, the quotient of 11 divided by 3 is 01 because 11 is 3 written in binary. Another example is the quotient of 1101 divided by 3 is 0100 because 1101 is 13 written binary and 0100 is 4 written in binary.