# CSE 322
# Winter Quarter 2003
# Assignment 1
# Due Friday, January 10

All solutions should be neatly written or type set. All major steps in proofs must be justified.

1. (10 points) This problem is designed to strengthen your ability to prove facts by induction. The reversal $w^R$ of a string $w$ can be defined recursively in the following way.

$$\begin{aligned} \epsilon^R &= \epsilon \\ (xa)^R &= ax^R \end{aligned}$$

   where $a \in \Sigma$.

   Prove the following:

   (a) For $a \in \Sigma$, $a^R = a$.

   (b) For $x$ and $y$ strings over $\Sigma$, $(xy)^R = y^R x^R$. For this your proof should be by induction on the length of $y$. You may use recursive definition of reversal and any basic facts about concatenations such as associativity and the identity properties of $\epsilon$.

2. (10 points) This problem is designed to build the background for analysis of finite automata. You may have heard of the pigeon hole principle before.

   **Pigeon Hole Principle:** If $f : S \to T$ and $S$ has more elements than $T$, then there are members $x$ and $y$ of $S$ such that $x \neq y$ and $f(x) = f(y)$. Put another way, if there are $n$ pigeons that are living in $m$ coops and $n > m$ then there are at least two pigeons that live in the same coop.

   Suppose we have a directed graph $G = (V, E)$ where $V$ has cardinality $n$. A path in $G$ is a sequence $(v_0, v_1, \ldots, v_k)$ of vertices such that for $0 \leq i < k$, $(v_i, v_{i+1})$ is in $E$. For each $i$

we say that $v_i$ is visited on the path $(v_0, v_1, \ldots, v_k)$. The path $(v_0, v_1, \ldots, v_k)$ is of length $k$, which is the number of edges traversed on the path.

Prove, using the Pigeon Hole Principle, that every path of length $n$ or longer visits some vertex at least twice.