CSE 322
Autumn Quarter 2003
Assignment 7
Due Friday, November 21, 2003

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Given context-free grammars $G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$, design context-free grammars G such that:

   (a) $L(G) = L(G_1)L(G_2)$ (concatenation),

   (b) $L(G) = L(G_1)^*$ (Kleene star),

   (c) $L(G) = L(G_1)^R$ (reversal).

2. (10 points) Consider the context free-grammar:

$$\begin{array}{lcl} S & \to & ASAS \mid A \mid \varepsilon \\ A & \to & 110 \mid \varepsilon \end{array}$$

   Use the method described in class to convert the grammar into Chomsky normal form. In this method do the steps in the following order: (i) add a new start symbol if ε is generated by the grammar, (ii) shorten productions whose right hand sides are longer than 2, (iii) remove ε-rules, (iv) remove unit rules, (v) make all right hand sides of length 2 into nonterminals.

3. (10 points) Let $G = (V, \Sigma, R, S)$.

   (a) A nonterminal A is *productive* if $A \Rightarrow_G^* w$ for some $w \in \Sigma^*$. That is, some terminal string can be generated from A. Design a closure algorithm for finding all the productive nonterminals in a grammar G.

   (b) Use the algorithm in part (a) as part of an algorithm for deciding if the language generated by a context-free grammar is empty.

   (c) Use the algorithm in part (a) to construct a context-free grammar G' such that $L(G) = L(G')$ and for all α, if $S \Rightarrow_{G'}^* \alpha$ then $\alpha \Rightarrow_{G'}^* w$ for some $w \in \Sigma^*$. That is, G' and G generate the same language and in G' every partial derivation can be eventually completed into the derivation of some terminal string

4. (10 points) Let $M_1 = (Q_1, \Sigma, \Gamma, \delta_1, q_1, F_1)$ be a PDA and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA.

   (a) Use a cross product construction to build a PDA M such that $L(M) = L(M_1) \cap L(M_2)$. Take particular care in defining the transition function for M. This shows that the context-free languages are closed under intersection with regular languages.

   (b) State a behavioral lemma for your construction that can be used to show $L(M) = L(M_1) \cap L(M_2)$.