CSE 322
Autumn Quarter 2003
Assignment 4
Due Friday, October 24, 2003

All solutions should be neatly written or type set. All major steps in proofs must be justified.

1. (10 points) In this problem you will study the relationships between prefixes, suffixes, and reversals. Recall that u is a *prefix* of x, if for some string y, $x = uy$. Similarly, v is a *suffix* of x, if for some string y, $x = yv$. Define the following operations on languages.

$$\text{Pre}(L) = \{u : u \text{ is a prefix of } x \text{ for some } x \in L\}$$
$$\text{Suff}(L) = \{v : v \text{ is a suffix of } x \text{ for some } x \in L\}$$

   (a) Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that accepts L, construct a DFA M' with the property that $L(M') = \text{Pre}(L)$.

   (b) Show that $\text{Suff}(L)$ is definable in terms the reversal of a language and the prefix of a language operations. (Hint: what I mean by definable is what I mean when I say that intersection is definable in terms of union and complement, using DeMorgans Law: $A \cap B = \overline{\overline{A} \cup \overline{B}}$.)

   (c) Explain, using (b) and other facts you know, why if L is accepted by a DFA, then $\text{Suff}(L)$ is accepted by a DFA.

2. (10 points) In this problem you can practice some of the constructions we are doing. Consider the regular expression $\alpha = (00 \cup 01)^*11$.

   (a) Carefully construct the equivalent NFA (a state diagram) that accepts the language defined by α. The construction is shown in the proofs of theorems 1.22, 1.23, and 1.24 in the book. Do not take any shortcuts.

(b) From the result in (a) above construct the equivalent NFA that has no ε-transitions. The main idea in the construction is to create a new transition on symbol a from state p to q if there is a sequence of ε-transitions from p to some state r, followed by a transition on symbol a from r to q. The set of final states has to be increased. Remove all the unreachable states.

3. (10 points) In this problem you will use Warshall's algorithm to determine if the language accepted by a finite automaton is finite or infinite. Given an NFA (without ε-transitions) $M = (Q, \Sigma, \delta, q_0, F)$ there is a natural directed graph $G_M = (Q, E)$ that simply represents the graphical structure of the state diagram. In this case $(i, j) \in E$ if $j \in \delta(i, a)$ for some $a \in \Sigma$. Using Warshall's algorithm we can find the reflexive, transitive closure of G_M, which we call $G^*_M = (Q, E^*)$ where $(i, j) \in E^*$ if and only if there is a path from i to j in G_M.

   First let me explain an algorithm for determining emptiness. Given $M = (Q, \Sigma, \delta, q_0, F)$ we want to determine if $L(M)$ is empty or not. The algorithm proceeds as follows:

   i. Construct $G_M = (Q, E)$.

   ii. Use Warshall's algorithm to compute $G^*_M = (Q, E^*)$.

   iii. $L(M)$ is non-empty if and only if $(i_0, j) \in E^*$ for some j which is a final state and where i_0 is the start state.

   The reason that this algorithm works is that $(i_0, j) \in E^*$ if and only if there is some path in G_M from i_0 to j, which holds if and only if there is some string w such that when M is started in state i_0 the machine M can consume w and end up in state j.

   Design and justify a high level algorithm like the one above to determine if $L(M)$ is infinite or not. Assume that M has no ε-transitions. Note that $L(M)$ is infinite if and only if there is some w such that starting in state i_0, a final state j can be reached by consuming w and there is some intermediate state k which is visited twice.