University of Washington
CSE 322: Introduction to Formal Models in Computer Science
Homework #5
Due: Friday, May 17, 2002, 10:30am

Spring 2002 May 8, 2002

Written homework is due at the *beginning* of class on the day specified. Any homework turned in after the deadline will be considered late. **Late homework policy:** You may turn in your homework after the deadline and before 5pm on the day it was due, but at a cost of a **20% penalty**. No homework will be accepted after 5pm on the due date.

Please staple all of your pages together (and order them according to the order of the problems below) and have your name on each page, just in case the pages get separated. Write legibly (or type) and organize your answers in a way that is easy to read. Neatness counts!

For each problem, make sure you have acknowledged all persons with whom you worked. Even though you are encouraged to work together on problems, the work you turn in is expected to be your own. When in doubt, invoke the *Gilligan's Island* rule (see the course organization handout) or ask the instructor.

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Regular problems (to be turned in) **:**

1. Using closure properties of regular languages, show that the language $L = \{ a^i b^j c^k \mid \text{if } i = 0, \text{ then } j \neq k \}$ is not regular.
2. For the context-free grammar G_4 given in Example 2.3 (page 95), show a parse tree for the string $a + a \times (a + a)$. Also, show each step of the leftmost derivation of the string.
3. Let $L = \{ 0^m 1^n \mid n \leq m \leq 2n \}$. Give a grammar G such that $L(G) = L$. Prove (formally) that the language of your grammar is L.
4. In the proof that all regular languages are also context-free given in lecture, there was an unproved claim that
$$\text{for all } i, j \text{ and } w \in \Sigma^*,\ R_i \Rightarrow^*_G w R_j \text{ if and only if } (q_i, w) \vdash^*_M (q_j, \varepsilon) .$$
Prove this claim by induction on $|w|$.
5. Give a PDA for the language $L = \{ a^m b^n c^p d^q \mid m + n = p + q, \text{ and } m, n, p, q \geq 0 \}$. (This is the same language as in problem #4 in Homework #4.) Do **not** use the context-free grammar to PDA construction in the book. Instead, directly construct such a PDA. You do not need to prove formally that your PDA accepts L, but instead provide an explanation for how your PDA works.
6. Give a PDA for the language in Exercise 2.6d. Do **not** use the context-free grammar to PDA construction in the book. Instead, directly construct such a PDA. You do not need to prove formally that your PDA accepts L, but instead provide an explanation for how your PDA works.

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Bonus Problem (optional):

1. Consider the language and context-free grammar in Suggested Problem #3. Here is how we would prove that $L(G) \subseteq L$.
   We first note that in each string $x \in (V \cup \Sigma)^*$, in any derivation of S to a string $w \in (V \cup \Sigma)^*$, there is at most one variable in x. (It is easy to prove this by induction on the length of the derivation.) This is because every rule in R replaces a variable with at most one variable. Now consider the following facts:

   Claim 1: For any $w \in (V \cup \Sigma)^*$, if $E \Rightarrow^*_G w$, then $w = b^n\ E\ c^n$ or $w = b^n c^n$, for some $n \geq 0$.
   Claim 2: For any $w \in (V \cup \Sigma)^*$, if $C \Rightarrow^*_G w$, then $w = a^m\ C\ c^m$ or $w = a^m c^m$, for some $m \geq 0$, or $w = a^m\ b^n\ E\ c^p$ or $w = a^m\ b^n\ c^p$, where $m + n = p$.
   Claim 3: For any $w \in (V \cup \Sigma)^*$, if $B \Rightarrow^*_G w$, then $w = b^n\ B\ d^n$ or $w = b^n d^n$, for some $n \geq 0$, or $w = b^n\ E\ c^p\ d^q$ or $w = b^n\ c^p\ d^q$, where $n = p + q$.
   Claim 4: For any $w \in (V \cup \Sigma)^*$, if $S \Rightarrow^*_G w$, then either
   (a) $w = a^m\ S\ d^m$, for some $m \geq 0$,
   (b) $w = a^m\ b^n\ B\ d^q$, where $m + n = q$,
   (c) $w = a^m\ C\ c^p\ d^q$, where $m = p + q$,
   (d) $w = a^m\ b^n\ E\ c^p\ d^q$, where $m + n = p + q$, or
   (e) $w = a^m\ b^n\ c^p\ d^q$, where $m + n = p + q$.

   Prove each of the claims. All of the claims need to be proved by induction on the length of the derivation. Claim 1 is easy to prove. Claims 2 and 3 are a little harder to prove, but you must use Claim 1 in the proofs. The proof of Claim 4 uses Claims 1 through 3. Then prove that $L(G) \subseteq L$ using Claim 4.

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Suggested problems (highly recommended, but not to be turned in) **:**

1. Show that $L = \{ a^n \mid n \text{ is prime} \}$ is not regular.
2. Show that for any DFA $M = (Q, \Sigma, \delta, q_0, F)$, M accepts an infinite language if and only if M accepts some string of length greater than or equal to $|Q|$ and less than $2\,|Q|$.
3. Once again, let us consider the language $L = \{ a^m\ b^n\ c^p\ d^q \mid m + n = p + q, \text{ and } m, n, p, q \geq 0 \}$. Now let G = (V, Σ, R, S) be a context-free grammar, where V = {S, B, C, E}, Σ = { a, b, c, d }, and R consists of the following rules:

(1) $S \to \varepsilon$	(6) $B \to b\ B\ d$	(9) $C \to a\ C\ c$
(2) $S \to a\ S\ d$	(7) $B \to b\ E\ c$	(10) $C \to b\ E\ c$
(3) $S \to b\ B\ d$	(8) $B \to \varepsilon$	(11) $C \to \varepsilon$
(4) $S \to a\ C\ c$		(12) $E \to b\ E\ c$
(5) $S \to b\ E\ c$		(13) $E \to \varepsilon$

   Prove that $L \subseteq L(G)$. You will need to show that if $w = a^m\ b^n\ c^p\ d^q$, where $m + n = p + q$, then there is a derivation from S to w in G. (That is, $S \Rightarrow^*_G w$.)

   (Showing the other half of the equality, that $L(G) \subseteq L$, is what the Bonus Problem is.)
4. For the languages in Exercises 2.4 and 2.6, directly construct PDAs that accept them.
5. 2.17.