CSE 322
Winter Quarter 2001
Assignment 9
Due Friday, March 9, 2001

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Use the pumping lemma to show that the language $\{0^n 1^{2n} 2^{3n} : n \geq 0\}$ is not context-free.

2. (20 points) Recall the definition of the prefix of a language in the alphabet Σ,

$$\text{Prefix}(L) = \{x : xy \in L \text{ for some } y \in \Sigma^*\}.$$

   In this problem you will give two different proofs of the fact that if L is context-free then so is Prefix(L).

   (a) The PDA Proof: Let $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a PDA. Construct a PDA M' such that $L(M') =$ Prefix$(L(M))$.

   (b) The Grammar Proof: Let $G = (V, \Sigma, R, S)$ be a Chomsky normal form grammar. Construct a context-free grammar G' such that $L(G') =$ Prefix$(L(G))$. Start with a Chomsky normal form grammar, because it is easier to reason about than a general context-free grammar. The grammar G' should have a new nonterminal A' for each nonterminal in G and there will be new rules about A'. The new nonterminal A' derives all prefixes of terminal strings w such that A derives w.

3. (10 points) Design a Turing machine (using a state diagram) which decides the language $\{0^n 1^n 2^n : n \geq 0\}$.