CSE 322

Introduction to Formal Models in Computer Science

Cocke-Kasami-Younger Algorithm example

Consider the following Chomsky Normal form grammar:

$$
\begin{array}{rcl}
S & \rightarrow & aT \mid aU \mid e \\
T & \rightarrow & Ub \mid b \\
U & \rightarrow & aT \mid UT
\end{array}
$$

We now show the computation of the tableau for the Cocke-Kasami-Younger algorithm on input $aaabbb$.

We begin with the single symbols:

6						b, T
5					b, T	
4				b, T		
3			a			
2		a				
1	a					
	1	2	3	4	5	6

Now we fill in the table for each entry just below the diagonal, looking for rules whose right hand side is composed of an element of the cell just to the left, followed by an element of the cell just above. The only right hand side that is of this form is aT.

6						b, T
5					b, T	$\emptyset$
4				b, T	$\emptyset$	
3			a	S, U		
2		a	$\emptyset$			
1	a	$\emptyset$				
	1	2	3	4	5	6

Now we fill in the next range. Note that the general form of the right-hand sides of rules for a cell one looks for involves a series of combinations beginning with the combination of the entry just to that cell's left with the highest entry in that cell's column and moving simulataneous leftward in the row and downward in the column.

In this case, most of the pairs involve empty cells ($\emptyset$) as one of the elements and so nothing can be generated. The combination of aU from the cells (2,2) and (3,4) together generate an S in cell (2,4). The combinations UT and Ub obtained from cells (3,4) and (5,5) generate U and T, respectively in entry (3,5).

6						b,T
5					b,T	$\emptyset$
4				b,T	$\emptyset$	$\emptyset$
3			a	S,U	U,T	
2		a	$\emptyset$	S		
1	a	$\emptyset$	$\emptyset$			
	1	2	3	4	5	6

For the next layer, UT and Ub are both obtained from cells (3,5) and (6,6) and generate U and T respectively in cell (3,6). Also, aT and aU both obtained from (2,2) and (3,5) generate U and S in cell (2,5).

6						b,T
5					b,T	$\emptyset$
4				b,T	$\emptyset$	$\emptyset$
3			a	S,U	U,T	U,T
2		a	$\emptyset$	S	S,U	
1	a	$\emptyset$	$\emptyset$	$\emptyset$		
	1	2	3	4	5	6

Now an aU obtained from (1,1) and (2,5) together generate an S in cell (1,5). An aT and aU obtained from (2,2) and (3,6) generate U and S in cell (2,6). Also, an Ub obtained from (2,5) and (6,6) generates a T in cell (2,6), a UT obtained from the same pair of cells is another way to generate S in (2,6).

6						b,T
5					b,T	$\emptyset$
4				b,T	$\emptyset$	$\emptyset$
3			a	S,U	U,T	U,T
2		a	$\emptyset$	S	S,U	S,T,U
1	a	$\emptyset$	$\emptyset$	$\emptyset$	S	
	1	2	3	4	5	6

Finally, an aT and an aU from cells (1,1) and (2,6) creates S and U in cell (1,6). Since S is in this cell the input $aaabbb$ is generated by the grammar.

6						b,T
5					b,T	$\emptyset$
4				b,T	$\emptyset$	$\emptyset$
3			a	S,U	U,T	U,T
2		a	$\emptyset$	S	S,U	S,T,U
1	a	$\emptyset$	$\emptyset$	$\emptyset$	S	S,U
	1	2	3	4	5	6