CSE 321: Discrete Structures
Assignment #3
April 12, 1999
due: Monday, April 19

1. Carefully prove the following implications.

   (a) $A \cup B = B \rightarrow A \subseteq B$

   (b) $A \cap B = A \rightarrow A \subseteq B$

2. For each of the following functions, state whether or not it is injective, and whether or not it is surjective. Justify your answers.

   (a) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = n + 1$.

   (b) $f : \mathbf{Z} \rightarrow \mathbf{Z}$, where $f(n) = n + 1$.

   (c) $f : \mathbf{R} \rightarrow \mathbf{R}$, where $f(n) = n + 1$.

   (d) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = \lfloor n/2 \rfloor$.

   (e) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = 2 \lfloor n/2 \rfloor$.

   (f) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-1, & \text{if } n \text{ is odd} \end{cases}$.

3. Suppose you graph a function $f : \mathbf{R} \rightarrow \mathbf{R}$. The fact that f is a function means that any straight vertical line will intersect the graph of f at exactly one point. What similar statement can you make about the graph of f if f is

   (a) injective?

   (b) surjective?

   (c) bijective?

4. Section 1.6, exercise 22.

5. Section 1.6, exercise 50. Be sure your graph extends into both positive and negative values of x.

6. Section 1.6, exercise 58. (Hint: For both halves of your proof, you may find it easier to prove the contrapositive.)