CSE 321: Discrete Structures
Assignment #3
October 11, 1999
due: Monday, October 18

1. Carefully prove the following implications.
   (a) $(A \cup B = B) \rightarrow (A \subseteq B)$
   (b) $(A \cap B = A) \rightarrow (A \subseteq B)$

2. For each of the following functions, state whether or not it is injective, and whether or not it is surjective. Justify your answers.
   (a) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = n^2$.
   (b) $f : \mathbf{Z} \rightarrow \mathbf{N}$, where $f(n) = n^2$.
   (c) $f : \mathbf{R} \rightarrow \mathbf{R}$, where $f(n) = 3n + 7$.
   (d) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = \lceil n/3 \rceil$.
   (e) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = 3 \lceil n/3 \rceil$.
   (f) $f : \mathbf{N} \rightarrow \mathbf{N}$, where $f(n) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-1, & \text{if } n \text{ is odd} \end{cases}$.

3. Suppose you graph a function $f : \mathbf{R} \rightarrow \mathbf{R}$. The fact that f is a function means that any straight vertical line will intersect the graph of f at exactly one point. What similar statement can you make about the graph of f if f is
   (a) injective?
   (b) surjective?
   (c) bijective?

4. Section 1.6, exercise 22.

5. Section 1.6, exercise 50. Be sure your graph extends into both positive and negative values of x.

6. Section 1.6, exercise 58. (Hint: For both halves of your proof, you may find it easier to prove the contrapositive. An informal justification in English is sufficient, because we haven't built up enough tools yet to do this one as a formal proof. In particular, it will be helpful for you to use the "pigeonhole principle", Theorem 1 in Section 4.2.)