Section 04: Formal Proofs and English Translation

1. Divisibility

(a) Circle the statements below that are true. Recall for $a, b \in \mathbb{Z}$: $a \mid b$ if and only if $\exists k \in \mathbb{Z}$ such that $b = ka$.

(i) $1 \mid 3$

(ii) $3 \mid 1$

(iii) $2 \mid 2018$

(iv) $-2 \mid 12$

(v) $1 \cdot 2 \cdot 3 \cdot 4 \mid 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$

(b) Circle the statements below that are true. Recall for $a, b, m \in \mathbb{Z}$ and $m > 0$: $a \equiv b \pmod{m}$ if and only if $m \mid (a - b)$.

(i) $-3 \equiv 3 \pmod{3}$

(ii) $0 \equiv 9000 \pmod{9}$

(iii) $44 \equiv 13 \pmod{7}$

(iv) $-58 \equiv 707 \pmod{5}$

(v) $58 \equiv 707 \pmod{5}$

2. Just The Setup

For each of these statements,

- Translate the sentence into predicate logic.
- Write the first few sentences and last few sentences of the English proof.

(a) The product of an even integer and an odd integer is even.

(b) There is an integer x s.t. $x^2 > 10$ and $3x$ is even.

(c) For every integer n, there is a prime number p greater than n.

3. Modular Arithmetic

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers greater than 0, then $a = b$ or $a = -b$.

(b) Prove that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.

4. Become a Mod God

Prove from definitions that for integers a, b, c, d and positive integer m, if $a \equiv b$ (mod m) and $c \equiv d$ (mod m), then $a - c \equiv b - d$ (mod m).

5. Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$.

6. Even Numbers, Odd Results!

For any integer j, if $3j + 1$ is even, then j is odd

(a) Write the predicate logic of this claim

Odd(x) := x is $2k + 1$, for some integer k
Even(x) := x is $2k$, for some integer k

(b) Write the contrapositive of this claim

(c) Determine which claim is easier to prove, then prove it!

7. The Trifecta

Consider the following proposition: For each integer a, if 3 divides a^2, then 3 divides a

(a) Write the contrapositive of this proposition as a sentence:

(b) Prove the proposition by proving its contrapositive.
Hint: Consider using cases based on the Division Algorithm using the remainder for "division by 3." There will be two cases!