

We're just answering
your questions today

Proof $a, b \in \mathbb{Z}$, p prime. If $ab \equiv 0 \pmod{p}$

then $a \equiv 0 \pmod{p}$ or $b \equiv 0 \pmod{p}$

Hint: If $p \mid xy \rightarrow p \mid x$ or $p \mid y$

— or —

unique (only one)

exactly two

→ there is a thing and
all other things are not a second of
there is a unique prime even integer.

neg

$$\exists x (\text{Prime}(x) \wedge \text{Even}(x)) \wedge \forall y (\text{Prime}(y) \wedge \text{Even}(y)) \rightarrow x=y$$

Handwritten annotations: The expression is underlined in red. A green circle highlights the universal quantifier $\forall y$. A green arrow points from the top right to the universal quantifier. A red arrow points from the universal quantifier to the implication's consequent $x=y$. A large green arrow curves from the left side of the expression back to the universal quantifier.

$$\exists x \exists y (\text{w}(x) \wedge \text{w}(y) \wedge x \neq y) \wedge \forall z (\text{w}(z) \rightarrow [x=z \vee y=z])$$

Handwritten annotations: The expression is written in purple. A green circle highlights the $x \neq y$ part. A green arrow points from the universal quantifier $\forall z$ to the implication's consequent $[x=z \vee y=z]$. A green arrow points from the universal quantifier $\forall z$ to the existential quantifier $\exists x$.

$$\exists x (P(x) \wedge E(x)) \wedge \forall y ([P(y) \wedge E(y)] \rightarrow y=x)$$

$$\forall x (\neg P(x) \vee \neg E(x)) \vee \neg [\forall y (P(y) \wedge E(y) \rightarrow y=x)]$$

$$\forall x (\neg P(x) \vee \neg E(x)) \vee \exists y (P(y) \wedge E(y) \wedge y \neq x)$$

$$\forall x ([P(x) \wedge E(x)] \rightarrow \exists y (\dots))$$

~~$\neg(a \rightarrow b)$~~

$\neg(\neg a \vee b)$

~~$a \wedge \neg b$~~

quantifiers
contrapositives

~~$\forall x (P(x) \rightarrow Q(x))$~~

$(\forall x (P(x))) \rightarrow (\exists x Q(x))$

1. Define $P(n)$
 - returns T/F
 - make sure it depends on n

2 Base Case(s)

3. ~~III~~ Suppose $P(n)$ holds for
 $n = b, b+1, \dots, k$ for an
arbitrary k last base case

4. IS ... $P(k+1)$

5 Conclusion