Full outline

1. Suppose for the sake of contradiction that L is regular. Then there is some DFA M that recognizes L.

2. Let S be [fill in with an infinite set of prefixes].

3. Because the DFA is finite and S is infinite, there are two (different) strings x, y in S such that x and y go to the same state when read by M *[you don't get to control x, y other than having them not equal and in S]*

4. Consider the string z [argue exactly one of xz, yz will be in L]

5. Since x, y both end up in the same state, and we appended the same z, both xz and yz end up in the same state of M. Since $xz \in L$and $yz \notin L$, M does not recognize L. But that's a contradiction!

6. So L must be an irregular language.

Countable

Countable

The set A is countable iff there is an injection from A to $\mathbb{N}$, Equivalently, A is countable iff it is finite or there is a bijection from A to $\mathbb{N}$

$\mathbb{N}, \mathbb{Z}$, $\{x: x$ is an even integer$\}$ are all countable.

To build a bijection from $\boldsymbol{A}$ to $\mathbb{N}$, just list all the elements!

Proof that [0,1) is not countable

Suppose, for the sake of contradiction, that there is a list of them:

Number	Digits after decimal	0	1	2	3	4	5	6	7	…
$f(0)$	0.	3	3	3	3	3	[illegible]	[illegible]	[illegible]	[illegible]
$f(1)$	0.	2	7	2	7	2	[illegible]	[illegible]	[illegible]	[illegible]
$f(2)$	0.	1	4	1	5	9	[illegible]	[illegible]	[illegible]	[illegible]
$f(3)$	0.	2	2	2	2	2	[illegible]	[illegible]	[illegible]	[illegible]
$f(4)$	0.	1	2	3	4	5	6	7	8	…
$f(5)$	0.	9	8	7	6	5	4	3	2	…
$f(6)$	0.	8	2	7	6	4	5	7	4	…
$f(7)$	0.	5	9	4	2	7	5	1	7	…
…	…	…	…	…	…	…	…	…	…	…

Goal: find a real number between 0 and 1 that isn't on our table.
(contradiction to bijection)

Bijection

One-to-one (aka injection)

A function f is one-to-one iff $\forall a \forall b (f(a) = f(b) \rightarrow a = b)$

Onto (aka surjection)

A function $f: A \rightarrow B$ is onto iff $\forall b \in B \exists a \in A (b = f(a))$

Bijection

A function $f: A \rightarrow B$ is a bijection iff f is one-to-one and onto

A bijection maps every element of the domain to exactly one element of the co-domain, and every element of the domain to exactly one element of the domain.