Nested Quantifiers

Let our domain of discourse be $\{A, B, C, D, E\}$

And our proposition $P(x, y)$ be given by the table.

What should we look for in the table?

$\exists x \forall y P(x, y)$

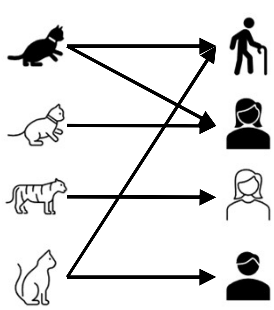
$\forall x \exists y P(x, y)$

y (columns), x (rows)

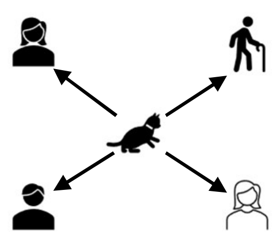
$P(x,y)$	A	B	C	D	E
A	T	T	T	T	T
B	T	F	F	T	F
C	F	T	F	F	F
D	F	F	F	F	T
E	F	F	F	T	F

Try it yourselves

Every cat loves some human.



There is a cat that loves every human.



Let your domain of discourse be mammals.
Use the predicates `Cat`(x), `Dog`(x), and `Loves`(x, y) to mean x loves y.

Our First Direct Proof

Definitions
$\text{Even}(x) \coloneqq \exists k(x = 2k)$

Prove: "For all integers x, if x is even, then x^2 is even." $\forall x\left(\text{Even}(x) \rightarrow \text{Even}(x^2)\right)$

Proof: Let x be an arbitrary integer. Suppose that x is even.

Integer

We need a basic starting point to be able to prove things.

Objects to work with.

An integer: is any real number with no fractional part.

Some definitions to analyze

Even	Odd
`Even(x)` := An integer, x, is even if and only if there is an integer k such that $x = 2k$.	**`Odd(x)` := An integer, x, is odd if and only if there is an integer k such that $x = 2k + 1$.**