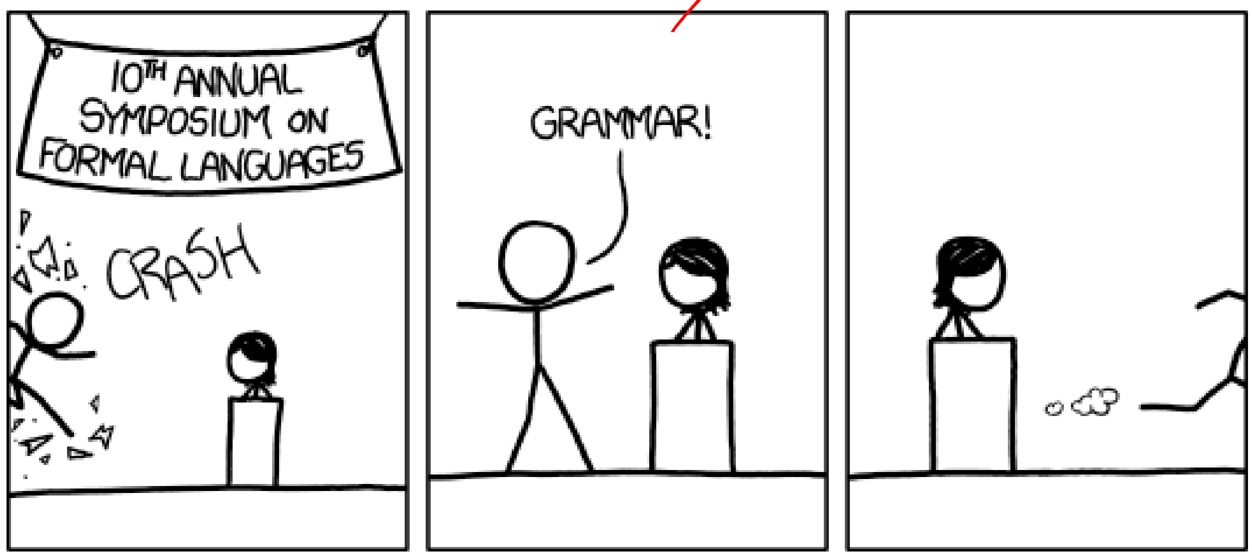
CSE 311: Foundations of Computing

Lecture 19: Context-Free Grammars

Do not ERASE



3 edition both

[Audience looks around]
"What is going on? There must be some context we're missing"

Midterm Review Session Q&A

Starting 4:30 pm on Zoom

Midterm Wed in class

Last class: Languages: Sets of Strings

- Subsets of strings are called *languages*
- Examples:
  - $\Sigma^* =$ All strings over alphabet Σ
  - Palindromes over Σ
  - Binary strings that don't have a 0 after a 1
  - Binary strings with an equal # of 0's and 1's
  - Legal variable names in Java/C/C++
  - Syntactically correct Java/C/C++ programs
  - Valid English sentences

Last class: Regular Expressions

Regular expressions over Σ

- **Basis:**

  ε is a regular expression (could also include $\varnothing$)

  a is a regular expression for any $a \in \Sigma$

- **Recursive step:**

  If **A** and **B** are regular expressions then so are:

  $\mathbf{A} \cup \mathbf{B}$

  AB

  A*

Last class: Regular Expression is a "pattern"

ε matches the **empty string**

a matches the one character string *a*

A $\cup$ **B** matches all strings that either **A** matches or **B** matches (or both)

AB matches all strings that have a first part that **A** matches followed by a second part that **B** matches

A* matches all strings that have any number of strings (even 0) that **A** matches, one after another

Yields a *language* = the set of strings matched by the regular expression

Last class: Examples

Regular Expression	Language
*001**	{00, 001, 0011, 00111, ...}
*0*1**	{Binary strings with any number of 0s followed by any number of 1s}
$(0 \cup 1)\ 0\ (0 \cup 1)\ 0$	{0000, 1000, 0010, 1010}
$(0^*1^*)^*$	{All binary strings}={0,1}*
$(0 \cup 1)^*$	{All binary strings}={0,1}*
$(0 \cup 1)^*\ 0110\ (0 \cup 1)^*$	{All binary strings containing substring 0110}

Regular Expressions in Practice

- Used to define the *tokens* of a programming language
  - legal variable names, keywords, etc.

- Used in `grep`, a program that does pattern matching searches in UNIX/LINUX

- We can use regular expressions in programs to process strings!

PHP
Perl

Regular Expressions in Java

```
Pattern p = Pattern.compile("a*b");
Matcher m = p.matcher("aaaaab");
boolean b = m.matches();
```

[01]	a 0 or a 1	**^** start of string	**$** end of string
[0-9]	any single digit	**\\.** period	**\\,** comma **\\-** minus
.	any single character		
ab	a followed by b	**(AB)**	
(a|b**)**	a or b	**(A ∪ B)**	
a**?**	zero or one of a	**(A ∪ ε)**	
a*****	zero or more of a	**A***	
a**+**	one or more of a	**AA***	

- e.g. **^[\-+]?[0-9]*(\.|\,)?[0-9]+$**

  General form of decimal number e.g. 9.12 or -9,8 (Europe)

Examples

- All binary strings that have an even # of 1's

0^* ~~wrong~~

$(0^* 1 0^* 1 0^*)^*$

$(0^* (1 0^* 1)^*)^*$

$(0 \cup (1 0^* 1))^*$

$(0^* 1 1 0^*)^* \cup 0^*$

$0^* (1 0^* 1 0^*)^*$

~~divided~~

divided

dul

Examples

- All binary strings that have an even # of 1's

  e.g., $0^* (10^*10^*)^*$

Examples

- All binary strings that have an even # of 1's

  e.g., *0* (10*10*)**

- All binary strings that *don't* contain 101

generate

101

110100

101

0

101

$(0^*11^*0^*)^*$

$(0 \cup (1000^*1) \cup (11))^*$

$(0^* \cup 1^* \cup (100^*1))^*$

$(1^*((00)^* \cup 000^*)1^*)^*$

$(0^*((1^*000^*1^*) \cup 1^*))^*0^*$

Examples

- All binary strings that have an even # of 1's

  e.g., $0^* (10^*10^*)^*$

- All binary strings that *don't* contain 101

  e.g., $0^* (1 \cup 000^*)^* 0^*$

  at least two 0s between 1s

Limitations of Regular Expressions

- Not all languages can be specified by regular expressions
- Even some easy things like
  - Palindromes
  - Strings with equal number of 0's and 1's
- But also more complicated structures in programming languages
  - Matched parentheses
  - Properly formed arithmetic expressions
  - etc.

Context-Free Grammars

- A Context-Free Grammar (CFG) is given by a finite set of substitution rules involving
  - Alphabet Σ of *terminal symbols* that can't be replaced
  - A finite set $\mathbf{V}$ of *variables* that can be replaced
  - One variable, usually $\mathbf{S}$, is called the *start symbol*

- The substitution rules involving a variable $\mathbf{A}$, written as

$$\mathbf{A} \rightarrow w_1 \mid w_2 \mid \cdots \mid w_k$$

where each w_i is a string of variables and terminals
  - that is $w_i \in (\mathbf{V} \cup \Sigma)^*$

How CFGs generate strings

- Begin with "**S**"

- If there is some variable **A** in the current string, you can replace it by one of the w's in the rules for **A**
  - $\mathbf{A} \rightarrow w_1 \mid w_2 \mid \cdots \mid w_k$
  - Write this as $x\mathbf{A}y \Rightarrow xwy$
  - Repeat until no variables left

- The set of strings the CFG describes are all strings, containing no variables, that can be *generated* in this manner after a finite number of steps

Example Context-Free Grammars

Example: $S \rightarrow 0S \mid S1 \mid \varepsilon$

Any number of 0's followed by any # of 1's

$0^* 1^*$

$S \Rightarrow 0S \Rightarrow 00S \Rightarrow 000S$

$\Rightarrow 000S1$

$\Rightarrow 0000S1$

$\Rightarrow 0000S11$

$\Rightarrow 000011$

Example Context-Free Grammars

Example: $\mathbf{S} \rightarrow 0\mathbf{S} \mid \mathbf{S}1 \mid \varepsilon$

0*1*

Example Context-Free Grammars

Example: $\mathbf{S} \rightarrow 0\mathbf{S} \mid \mathbf{S}1 \mid \varepsilon$

0*1*

Example: $\mathbf{S} \rightarrow 0\mathbf{S}0 \mid 1\mathbf{S}1 \mid 0 \mid 1 \mid \varepsilon$

binary palindromes

Example Context-Free Grammars

Example: $\mathbf{S} \rightarrow 0\mathbf{S} \mid \mathbf{S}1 \mid \varepsilon$

0*1*

Example: $\mathbf{S} \rightarrow 0\mathbf{S}0 \mid 1\mathbf{S}1 \mid 0 \mid 1 \mid \varepsilon$

The set of all binary palindromes

Example Context-Free Grammars

Grammar for $\{0^n 1^n : n \geq 0\}$

(i.e., matching *0*1 but with same number of 0's and 1's)**

$\varepsilon, 01, 0011, 000111, 00001111, \ldots$

$S \rightarrow 0S1 \mid 01 \mid \varepsilon$

Example Context-Free Grammars

Grammar for $\{0^n 1^n : n \geq 0\}$

(i.e., matching *0*1 but with same number of 0's and 1's)**

$$\mathbf{S} \rightarrow 0\mathbf{S}1 \mid \varepsilon$$

$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 0011$

Example Context-Free Grammars

Grammar for $\{0^n 1^n : n \geq 0\}$

(i.e., matching *0*1 but with same number of 0's and 1's)**

$$\mathbf{S} \rightarrow 0\underbrace{\mathbf{S}1}\ |\ \varepsilon$$

Grammar for $\{0^n 1^{2n} : n \geq 0\}$

011

001111

$$S \rightarrow 0S11\ |\ \varepsilon$$

Example Context-Free Grammars

Grammar for $\{0^n 1^n : n \geq 0\}$

(i.e., matching *0*1** but with same number of 0's and 1's)

$$\mathbf{S} \rightarrow 0\mathbf{S}1 \mid \varepsilon$$

Grammar for $\{0^n 1^{2n} : n \geq 0\}$

$$\mathbf{S} \rightarrow 0\mathbf{S}11 \mid \varepsilon$$

Example Context-Free Grammars

Grammar for $\{0^n1^n : n \geq 0\}$

(i.e., matching *0*1** but with same number of 0's and 1's)

$$S \rightarrow 0S1 \mid \varepsilon$$

Grammar for $\{0^n1^{n+1}0 : n \geq 0\}$

$\{0^n 1^n 10 : n \geq 0\}$

$S \rightarrow A10$

$A \rightarrow 0A1 \mid \varepsilon$

Example Context-Free Grammars

Grammar for $\{0^n 1^n : n \geq 0\}$

(i.e., matching ***0*1**** but with same number of 0's and 1's)

$$\mathbf{S} \rightarrow 0\mathbf{S}1 \mid \varepsilon$$

Grammar for $\{0^n 1^{n+1} 0 : n \geq 0\}$

$$\mathbf{S} \rightarrow A10$$

$$\mathbf{A} \rightarrow 0\mathbf{A}1 \mid \varepsilon$$

Example Context-Free Grammars

Example: $S \rightarrow (S) \mid SS \mid \varepsilon$

Example Context-Free Grammars

Example: $S \rightarrow (S) \mid SS \mid \varepsilon$

The set of all strings of matched parentheses