Section 4

CSE 311 - Sp 2022

Administrivia

Announcements and Reminders

- HW3 due yesterday 10PM on Gradescope
  - Final late due date is Saturday 4/23 @ 10pm
- HW2 grades out now
  - Regrade requests are open for one week
  - If you think your work may have been graded incorrectly, please submit a regrade request!
- HW4 is out!
  - Due Wednesday 4/27 @ 10pm
- Check the course website for OH times!
  - Friday, we have extended Allie & Sandy's OH to be from 4:30-6:20, and we'll be offering some targeted help during the first hour in particular, going through some of the questions from the new HW assignment and focusing on understanding what the question is asking, where you should start, and where you are trying to go

References

- Helpful reference sheets can be found on the course website!
  - https://courses.cs.washington.edu/courses/cse311/22sp/resources/
- How to LaTeX (found on Assignments page of website):
  - https://courses.cs.washington.edu/courses/cse311/22sp/assignments/HowToLaTeX.pdf
- Set Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/22sp/resources/reference-sets.pdf
- Number Theory Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/22sp/resources/reference-number-theory.pdf
- Plus more!

Proof Skeleton

Doing a Proof

- Don’t just jump right in!

- Look at the statement, and make sure you know:
    - What every word in the statement means
    - What the statement as a whole means
    - **Where to start**
    - **What your target is**

- It can help to see if you can first write the statement in predicate logic. Then, write your proof!

- Some Helpful Tips for English Proofs:
    - Start by introducing your assumptions
    - Introduce variables with “let”
    - Introduce assumptions with “suppose”
    - Always state what type your variable is
    - Don’t just use “algebra” as a reason, actually explain what’s going on

Problem 2 - Just the Setup

For each of these statements,

- Translate the sentence into predicate logic.
- Write the first few sentences and last few sentences of the English proof.
- Write the first few and last few steps of an inference proof of the statement (you do not need to write the middle – just enough to introduce all givens and assumptions and the conclusion at the end)

(a) The product of an even integer and an odd integer is even.

(b) There is an integer x such that $x^2 > 10$ and 3x is even.

(c) For every integer n, there is a prime number p greater than n.

(d) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$ for any sets A, B, C.

Work on parts (b) and (c) with the people around you, and then we'll go over it together!

Problem 2 - Just the Setup

(b) There is an integer x such that $x^2 > 10$ and 3x is even.

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$\exists x[\text{GreaterThan10}(x^2) \wedge \text{Even}(3x)]$

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Consider $x = 6$.

. . . .

Problem 2 - Just the Setup

(b) There is an integer x such that $x^2 > 10$ and 3x is even.

$\exists x[\text{GreaterThan10}(x^2) \land \text{Even}(3x)]$

Consider $x = 6$.

. . . .

Then there exists some integer k such that $3 \cdot 6 = 2k$.

Problem 2 - Just the Setup

(b) There is an integer x such that $x^2 > 10$ and 3x is even.

$\exists x[\text{GreaterThan10}(x^2) \wedge \text{Even}(3x)]$

Consider $x = 6$.

. . . .

Then there exists some integer k such that $3 \cdot 6 = 2k$.

So $6^2 > 10$ and $3 \cdot 6$ is even.

Hence, 6 is the desired x.

Problem 2 - Just the Setup

(b) There is an integer x such that $x^2 > 10$ and 3x is even.

$\exists x[\text{GreaterThan10}(x^2) \wedge \text{Even}(3x)]$

. . .

?. $\text{GreaterThan10}(6^2)$ [Definition of GreaterThan10]

. . .

?. $\exists k[3 \cdot 6 = 2k]$ [?]

?. $\text{Even}(3 \cdot 6)$ [Definition of Even]

?. $\text{GreaterThan10}(6^2) \wedge \text{Even}(3 \cdot 6)$ [Intro $\wedge$]

?. $\exists x[\text{GreaterThan10}(x^2) \wedge \text{Even}(3x)]$ [Intro $\exists$]

Problem 2 - Just the Setup

(c) For every integer n, there is a prime number p greater than n.

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Problem 2 - Just the Setup

(c) For every integer n, there is a prime number p greater than n.

$\forall x \exists y[\text{Prime}(y) \wedge \text{GreaterThan}(y, x)]$

Let x be an arbitrary integer.

Problem 2 - Just the Setup

(c) For every integer n, there is a prime number p greater than n.

$\forall x \exists y[\mathrm{Prime}(y) \wedge \mathrm{GreaterThan}(y, x)]$

Let x be an arbitrary integer.
Consider y = p (this p is a specific prime)
. . . .

Problem 2 - Just the Setup

(c) For every integer n, there is a prime number p greater than n.

$\forall x \exists y[Prime(y) \wedge GreaterThan(y, x)]$

Let x be an arbitrary integer.
Consider y = p (this p is a specific prime)
. . . .
So p is prime and p > x.

Problem 2 - Just the Setup

(c) For every integer n, there is a prime number p greater than n.

$\forall x \exists y[\text{Prime}(y) \land \text{GreaterThan}(y, x)]$

Let x be an arbitrary integer.
Consider y = p (this p is a specific prime)
. . . .
So p is prime and p > x.
Since x was arbitrary, we have that every integer has a prime number that is greater than it.

Problem 2 - Just the Setup

(c) For every integer n, there is a prime number p greater than n.

$\forall x \exists y[\text{Prime}(y) \land \text{GreaterThan}(y, x)]$

1. Let a be an arbitrary object	–
. . .	
?. Prime(b)	[Definition of Prime]
. . .	
?. GreaterThan(b, a)	[Definition of GreaterThan]
?. Prime(b) $\land$ GreaterThan(b, a)	[Intro $\land$]
?. $\exists y$[Prime(y) $\land$ GreaterThan(y, a)]	[Intro $\exists$]
?. $\forall x \exists y$[Prime(y) $\land$ GreaterThan(y, x)]	[Intro $\forall$]

Sets

Set Review

- A set is an **unordered** group of **distinct** elements
  - Set variable names are capital letters, with lower-case letters for elements

- Set Notation:
  - $a \in A$: “a is in A” or “a is an element of A”
  - $A \subseteq B$: “A is a subset of B”, every element of A is also in B
  - $\varnothing$: “empty set”, a unique set containing no elements
  - $\mathscr{P}(A)$: “power set of A”, the set of all subsets of A including the empty set and A itself

- Set Operator Definitions
  - Subset: $A \subseteq B \equiv \forall x\, (x \in A \rightarrow x \in B)$
  - Equality: $A = B \equiv \forall x\, (x \in A \leftrightarrow x \in B) \equiv A \subseteq B \land B \subseteq A$
  - Union: $A \cup B = \{x: x \in A \lor x \in B\}$
  - Intersection: $A \cap B = \{x: x \in A \land x \in B\}$
  - Complement: $\bar{A} = \{x: x \notin A\}$
  - Difference: $A \setminus B = \{x: x \in A \land x \notin B\}$
  - Cartesian Product: $A \times B = \{(a,b): a \in A \land b \in B\}$

Problem 3 - How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say ∞.

(a) $A = \{1, 2, 3, 2\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \dots\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathscr{P}(\{\varnothing\})$

Work on this problem with the people around you, and then we'll go over it together!

Problem 3 - How Many Elements?

(a) $A = \{1, 2, 3, 2\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$

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(f) $F = \mathscr{P}(\{\varnothing\})$

Problem 3 - How Many Elements?

(a) $A = \{1, 2, 3, 2\}$ $3, A = \{1, 2, 3\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

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Problem 3 - How Many Elements?

(a) $A = \{1, 2, 3, 2\}$ $3,\ A = \{1, 2, 3\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$ $2,\ B = \{\varnothing, \{\varnothing\}\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathscr{P}(\{\varnothing\})$

Problem 3 - How Many Elements?

(a) $A = \{1, 2, 3, 2\}$ $3, A = \{1, 2, 3\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \dots\}$ $2, B = \{\varnothing, \{\varnothing\}\}$

(c) $C = A \times (B \cup \{7\})$ $9, C = \{1, 2, 3\} \times \{7, \varnothing, \{\varnothing\}\}$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathscr{P}(\{\varnothing\})$

Problem 3 - How Many Elements?

(a) $A = \{1, 2, 3, 2\}$ $3, A = \{1, 2, 3\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$ $2, B = \{\varnothing, \{\varnothing\}\}$

(c) $C = A \times (B \cup \{7\})$ $9, C = \{1, 2, 3\} \times \{7, \varnothing, \{\varnothing\}\}$

(d) $D = \varnothing$ 0

(e) $E = \{\varnothing\}$

(f) $F = \mathscr{P}(\{\varnothing\})$

Problem 3 - How Many Elements?

(a) $A = \{1, 2, 3, 2\}$ 3, $A = \{1, 2, 3\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$ 2, $B = \{\varnothing, \{\varnothing\}\}$

(c) $C = A \times (B \cup \{7\})$ 9, $C = \{1, 2, 3\} \times \{7, \varnothing, \{\varnothing\}\}$

(d) $D = \varnothing$ 0

(e) $E = \{\varnothing\}$ 1

(f) $F = \mathscr{P}(\{\varnothing\})$

Problem 3 - How Many Elements?

(a) $A = \{1, 2, 3, 2\}$ 3, $A = \{1, 2, 3\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$ 2, $B = \{\varnothing, \{\varnothing\}\}$

(c) $C = A \times (B \cup \{7\})$ 9, $C = \{1, 2, 3\} \times \{7, \varnothing, \{\varnothing\}\}$

(d) $D = \varnothing$ 0

(e) $E = \{\varnothing\}$ 1

(f) $F = \mathscr{P}(\{\varnothing\})$ 2, $F = \{\varnothing, \{\varnothing\}\}$

Problem 4 - Set = Set

Prove the following set identities. Write both a formal inference proof and an English proof.

(a) Let the universal set be $\mathscr{U}$. Prove $A \cap \bar{B} \subseteq A \setminus B$ for any sets A, B.

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

Work on part (b) with the people around you, and then we'll go over it together!

Problem 4 - Set = Set

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Let x be an arbitrary element of $(A \cap B) \times C$.

…

Since x was an arbitrary element of $(A \cap B) \times C$ we have proved that $(A \cap B) \times C \subseteq A \times (C \cup D)$ as required.

Problem 4 - Set = Set

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

Let x be an arbitrary element of $(A \cap B) \times C$.
Then, by definition of Cartesian product, x must be of the form (y, z) where $y \in A \cap B$ and $z \in C$.
…
Since x was an arbitrary element of $(A \cap B) \times C$ we have proved that $(A \cap B) \times C \subseteq A \times (C \cup D)$ as required.

Problem 4 - Set = Set

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

Let x be an arbitrary element of $(A \cap B) \times C$.
Then, by definition of Cartesian product, x must be of the form (y, z) where $y \in A \cap B$ and $z \in C$.
Since $y \in A \cap B$, $y \in A$ and $y \in B$ by definition of $\cap$; in particular, all we care about is that $y \in A$.
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Since x was an arbitrary element of $(A \cap B) \times C$ we have proved that $(A \cap B) \times C \subseteq A \times (C \cup D)$ as required.

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(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

Let x be an arbitrary element of $(A \cap B) \times C$.
Then, by definition of Cartesian product, x must be of the form (y, z) where $y \in A \cap B$ and $z \in C$.
Since $y \in A \cap B$, $y \in A$ and $y \in B$ by definition of $\cap$; in particular, all we care about is that $y \in A$.
Since $z \in C$, by definition of $\cup$, we also have $z \in C \cup D$.
…
Since x was an arbitrary element of $(A \cap B) \times C$ we have proved that $(A \cap B) \times C \subseteq A \times (C \cup D)$ as required.

Problem 4 - Set = Set

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

Let x be an arbitrary element of $(A \cap B) \times C$.
Then, by definition of Cartesian product, x must be of the form (y, z) where $y \in A \cap B$ and $z \in C$.
Since $y \in A \cap B$, $y \in A$ and $y \in B$ by definition of $\cap$; in particular, all we care about is that $y \in A$.
Since $z \in C$, by definition of $\cup$, we also have $z \in C \cup D$.
Therefore since $y \in A$ and $z \in C \cup D$, by definition of Cartesian product we have $x = (y, z) \in A \times (C \cup D)$.
Since x was an arbitrary element of $(A \cap B) \times C$ we have proved that $(A \cap B) \times C \subseteq A \times (C \cup D)$ as required.

Problem 4 - Set = Set

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

1. Let x be arbitrary	
2.1 $x \in (A \cap B) \times C$	Assumption
2.2 $(y, z) \in (A \cap B) \times C$	Def of $\times$
2.3 $y \in (A \cap B) \wedge z \in C$	Def of $\times$
2.4 $y \in (A \cap B)$	Elim $\wedge$
2.5 $y \in A \wedge y \in B$	Def of $\cap$
2.6 $y \in A$	Elim $\wedge$
2.7 $z \in C$	Elim $\wedge$
2.8 $z \in C \vee z \in D$	Intro $\vee$
2.9 $z \in (C \cup D)$	Def of $\cup$
2.10 $y \in A \wedge z \in (C \cup D)$	Intro $\wedge$
2.11 $(y, z) \in A \times (C \cup D)$	Def of $\times$
2.12 $x \in A \times (C \cup D)$	Def of $\times$
3. $x \in (A \cap B) \times C \rightarrow x \in A \times (C \cup D)$	Direct Proof
4. $\forall x(x \in (A \cap B) \times C \rightarrow x \in A \times (C \cup D))$	Intro $\forall$
5. $(A \cap B) \times C \subseteq A \times (C \cup D)$	Def of $\subseteq$

Problem 5 - Set Equality

(a) Prove that $A \cap (A \cup B) = A$ for any sets A, B.

(b) Let $\mathscr{U}$ be the universal set. Show that $\overline{\overline{X}} = X$

Work on part (a) with the people around you, and then we'll go over it together!

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Let x be an arbitrary element of of $A \cap (A \cup B)$.

…

Since x was arbitrary, $A \cap (A \cup B) \subseteq A$.

Now let y be an arbitrary member of A. Then $y \in A$. So certainly $y \in A$ or $y \in B$.

…

Since y was arbitrary, $A \subseteq A \cap (A \cup B)$.

Therefore $A \cap (A \cup B) = A$, by containment in both directions.

Problem 5 - Set Equality

(a) Prove that $A \cap (A \cup B) = A$ for any sets A, B.

Let x be an arbitrary element of of $A \cap (A \cup B)$.
Then by definition of intersection, $x \in A$ and $x \in A \cup B$. So certainly, $x \in A$.
Since x was arbitrary, $A \cap (A \cup B) \subseteq A$.

Now let y be an arbitrary member of A. Then $y \in A$. So certainly $y \in A$ or $y \in B$.
…
Since y was arbitrary, $A \subseteq A \cap (A \cup B)$.

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Then by definition of intersection, $x \in A$ and $x \in A \cup B$. So certainly, $x \in A$.
Since x was arbitrary, $A \cap (A \cup B) \subseteq A$.

Now let y be an arbitrary member of A. Then $y \in A$. So certainly $y \in A$ or $y \in B$.
Then by definition of union, $y \in A \cup B$.
…
Since y was arbitrary, $A \subseteq A \cap (A \cup B)$.

Therefore $A \cap (A \cup B) = A$, by containment in both directions.

Problem 5 - Set Equality

(a) Prove that $A \cap (A \cup B) = A$ for any sets A, B.

Let x be an arbitrary element of of $A \cap (A \cup B)$.
Then by definition of intersection, $x \in A$ and $x \in A \cup B$. So certainly, $x \in A$.
Since x was arbitrary, $A \cap (A \cup B) \subseteq A$.

Now let y be an arbitrary member of A. Then $y \in A$. So certainly $y \in A$ or $y \in B$.
Then by definition of union, $y \in A \cup B$.
Since $y \in A$ and $y \in A \cup B$, by definition of intersection, $y \in A \cap (A \cup B)$.
Since y was arbitrary, $A \subseteq A \cap (A \cup B)$.

Therefore $A \cap (A \cup B) = A$, by containment in both directions.

That’s All, Folks!

Any questions?