

Try it!

Given: $p \vee q, (r \wedge s) \rightarrow \neg q, r$.
Show: $s \rightarrow p$

$$\text{Eliminate } \wedge \frac{A \wedge B}{\therefore A, B}$$

$$\text{Eliminate } \vee \frac{A \vee B, \neg A}{\therefore B}$$

$$\text{Intro } \wedge \frac{A; B}{\therefore A \wedge B}$$

$$\text{Intro } \vee \frac{A}{\therefore A \vee B, B \vee A}$$

$$\text{Direct Proof rule} \frac{A \Rightarrow B}{A \rightarrow B}$$

$$\text{Modus Ponens} \frac{P \rightarrow Q; P}{\therefore Q}$$

You can still use all the propositional logic equivalences too!

Proof Using Quantifiers

Suppose we know $\exists x P(x)$ and $\forall y [P(y) \rightarrow Q(y)]$. Conclude $\exists x Q(x)$.

$$\text{Intro } \exists \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\text{Eliminate } \exists \frac{\exists x P(x)}{\therefore P(c) \text{ for a fresh } c}$$

$$\text{Eliminate } \forall \frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$$

$$\text{Intro } \forall \frac{P(a); a \text{ is arbitrary}}{\therefore \forall x P(x)}$$

Arbitrary

In section, you said: $[\exists y \forall x P(x, y)] \rightarrow [\forall x \exists y P(x, y)]$. Let's prove it!!

Find The Bug

Let your domain of discourse be integers.

We claim that given $\forall x \exists y \text{ Greater}(y, x)$, we can conclude $\exists y \forall x \text{ Greater}(y, x)$

Where $\text{Greater}(y, x)$ means $y > x$

1. $\forall x \exists y \text{ Greater}(y, x)$ Given
2. Let a be an arbitrary integer --
3. $\exists y \text{ Greater}(y, a)$ Elim \forall (1)
4. $\text{Greater}(b, a)$ Elim \exists (2)
5. $\forall x \text{ Greater}(b, x)$ Intro \forall (4)
6. $\exists y \forall x \text{ Greater}(y, x)$ Intro \exists (5)