

# Section 02: Digital Logic and Equivalence Proofs

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## 1. Truth Tables

Write a truth table for each of the following:

(a)  $(p \oplus q) \vee (p \oplus \neg q)$

(b)  $(p \vee q) \rightarrow (p \oplus q)$

(c)  $p \leftrightarrow \neg p$

## 2. Non-equivalence

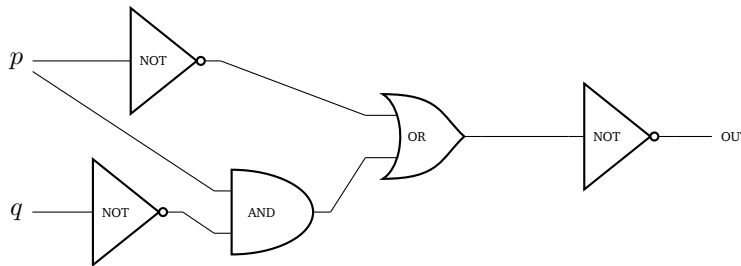
Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a)  $p \rightarrow q$  vs.  $q \rightarrow p$

(b)  $p \rightarrow (q \wedge r)$  vs.  $(p \rightarrow q) \wedge r$

## 3. Circuitous

Translate the following circuit into a logical expression.



## 4. Equivalences

Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

(a)  $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$  You may use the rule  $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ .

(b)  $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

## 5. Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

(a)  $\neg p \vee (\neg q \vee (p \wedge q))$

(b)  $\neg(p \vee (q \wedge p))$

## 6. Canonical Forms

Consider the boolean functions  $F(A, B, C)$  and  $G(A, B, C)$  specified by the following truth table:

$A$	$B$	$C$	$F(A, B, C)$	$G(A, B, C)$
1	1	1	1	0
1	1	0	1	1
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	0
0	0	1	0	1
0	0	0	1	0

(a) Write the DNF and CNF expressions for  $F(A, B, C)$ .

(b) Write the DNF and CNF expressions for  $G(A, B, C)$ .

## 7. Translate to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

(a) Every user has access to an electronic mailbox.

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

(c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

(d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

## 8. Translate to English

Translate these system specifications into English where  $F(p)$  is “Printer  $p$  is out of service”,  $B(p)$  is “Printer  $p$  is busy”,  $L(j)$  is “Print job  $j$  is lost,” and  $Q(j)$  is “Print job  $j$  is queued”. Let the domain be all printers and all print jobs.

(a)  $\exists p (F(p) \wedge B(p)) \rightarrow \exists j L(j)$

(b)  $(\forall j B(j)) \rightarrow (\exists p Q(p))$

(c)  $\exists j (Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

(d)  $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$