



xkcd.com/1153

Our First Proof and Digital Logic

CSE 311 Fall 2020
Lecture 3

Law of Implication

Implications are not totally intuitive.

AND/OR/NOT make more intuitive sense to me...

can we rewrite implications using just ANDs ORs and NOTs?

a	b	$a \rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

Seems like we might want $\neg(a \wedge \neg b)$

$$\neg a \vee b$$

Seems like a reasonable guess.

So is it true? Is $\neg a \vee b \equiv a \rightarrow b$?

Law of Implication

$$\neg a \vee b \equiv a \rightarrow b$$

a	b	$a \rightarrow b$	$\neg a$	$\neg a \vee b$	$(\neg a \vee b) \leftrightarrow (a \rightarrow b)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Warm-Up

“I go to the store, unless I order postmates.”

How would you translate “unless” into logic?

Warm-Up – Solution

“Dr. Evil will attack you with laser sharks, unless you pay him one-million dollars.”

If you pay Dr. Evil his one-million dollars, are you safe from the laser sharks?

This is Dr. Evil! He hasn't promised you that's his **full** list of demands.

If you do not pay Dr. Evil one-million dollars, then he will attack you with laser sharks.

Is the only promise you really have.

Warm-Up – Solution

“Dr. Evil will attack you with laser sharks, unless you pay him one-million dollars.”

a : Dr. Evil will attack you with laser sharks

b : You pay him one-million dollars

The last slide would give us: $\neg b \rightarrow a$

If Dr. Evil doesn't attack you with laser sharks, you must have paid him his million dollars, right? $\neg a \rightarrow b$ Are those both good translations?

Today

Our first proof!

Contrapositives and digital logic.

Announcements

Homework 1 Problem 6 clarified (download a new version of the pdf).

Office Hours start this week.

Homework Submissions

Make sure we can read what you submit.

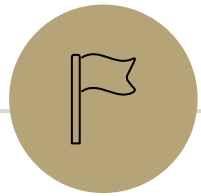
We can't spend 5 minutes per submission deciding if that's a p or a q .

Typesetting guarantees we can read it.

Microsoft Word's equation editor is now halfway decent!

LaTeX is the industry standard for typesetting (if you go to CS grad school, you'll use it for all your papers). Overleaf is the easiest way to get started.

Need to know the code for a symbol? [Detexify](#)! Word uses LaTeX codes...mostly...



Our First Proof



Last Time

We showed

DeMorgan's Laws:

$$\neg(a \vee b) \equiv \neg a \wedge \neg b \text{ and } \neg(a \wedge b) \equiv \neg a \vee \neg b$$

And the Law of Implication

$$a \rightarrow b \equiv \neg a \vee b$$

Properties of Logical Connectives

We will always give you this list!

For every propositions a, b, r the following hold:

- **Identity**
 - $a \wedge \text{T} \equiv a$
 - $a \vee \text{F} \equiv a$
- **Domination**
 - $a \wedge \text{F} \equiv \text{F}$
 - $a \vee \text{T} \equiv \text{T}$
- **Idempotent**
 - $a \vee a \equiv a$
 - $a \wedge a \equiv a$
- **Commutative**
 - $a \wedge b \equiv b \wedge a$
 - $a \vee b \equiv b \vee a$
- **Associative**
 - $(a \vee b) \vee r \equiv a \vee (b \vee r)$
 - $(a \wedge b) \wedge r \equiv a \wedge (b \wedge r)$
- **Distributive**
 - $a \wedge (b \vee r) \equiv (a \wedge b) \vee (a \wedge r)$
 - $a \vee (b \wedge r) \equiv (a \vee b) \wedge (a \vee r)$
- **Absorption**
 - $a \vee (a \wedge b) \equiv a$
 - $a \wedge (a \vee b) \equiv a$
- **Negation**
 - $a \vee \neg a \equiv \text{T}$
 - $a \wedge \neg a \equiv \text{F}$

Using Our Rules

WOW that was a lot of rules.

Why do we need them? Simplification!

Let's go back to the "law of implication" example.

p	b	$a \rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

When is the implication true? Just "or" each of the three "true" lines!

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b)$$

Also seems pretty reasonable

So is $(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv (\neg a \vee b)$

i.e. are these both alternative representations of $a \rightarrow b$?

Our First Proof

We could make another truth table (you should! It's a good exercise)

But we have another technique that is nicer.

Let's try that one

Then talk about why it's another good option.

We're going to give an iron-clad guarantee that:

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv \neg a \vee b$$

i.e. that this is another valid "law of implication"

Our First Proof

How do we write a proof?

It's not always plug-and-chug...we'll be highlighting strategies throughout the quarter.

To start with:

Make sure we know what we want to show...

Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$$

None of the rules look like this

Practice of Proof-Writing:

Big Picture... WHY do we think this might be true?

The last two "pieces" came from the $\equiv (\neg a \vee b)$ vacuous proof lines...maybe the " $\neg a$ " came from there? Maybe that **simplifies** down to $\neg a$

Let's apply a rule

$$(\neg a \wedge b) \vee (\neg a \wedge \neg b)$$

The law says:

$$a \wedge (b \vee r) \equiv (a \wedge b) \vee (a \wedge r)$$

$$(\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv \neg a \wedge (b \vee \neg b)$$

Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$$

None of the rules look like this

Practice of Proof-Writing:

Big Picture... WHY do we think this might be true?

The last two "pieces" came from the $\equiv (\neg a \vee b)$ vacuous proof lines...maybe the " $\neg a$ " came from there? Maybe that **simplifies** down to $\neg a$

Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)]$$

Set ourselves an intermediate goal.
Let's try to simplify those last two
pieces

Associative law

Connect up the things we're working on.

$$\equiv (\neg a \vee b)$$

Our First Proof

$$\begin{aligned}(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)]\end{aligned}$$

Set ourselves an intermediate goal.
Let's try to simplify those last two
pieces

Distributive law

We think $\neg a$ is important, let's isolate it.

$$\equiv (\neg a \vee b)$$

Our First Proof

$$\begin{aligned}(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge \text{T}]\end{aligned}$$

Set ourselves an intermediate goal.
Let's try to simplify those last two
pieces

Negation

Should make things simpler.

$$\equiv (\neg a \vee b)$$

Our First Proof

$$\begin{aligned}(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge \text{T}] \\ &\equiv (a \wedge b) \vee [\neg a]\end{aligned}$$

Set ourselves an intermediate goal.
Let's try to simplify those last two
pieces

Domination

Should make things simpler.

$$\equiv (\neg a \vee b)$$

Our First Proof

$$\begin{aligned}(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge \text{T}] \\ &\equiv (a \wedge b) \vee [\neg a] \\ &\equiv [\neg a] \vee (a \wedge b)\end{aligned}$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at $(\neg a \vee b)$

If we apply the distribution rule,

We'd get a $(\neg a \vee b)$

$$\equiv (\neg a \vee b)$$

Our First Proof

$$\begin{aligned}(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge \text{T}] \\ &\equiv (a \wedge b) \vee [\neg a] \\ &\equiv [\neg a] \vee (a \wedge b)\end{aligned}$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at $(\neg a \vee b)$

If we apply the distribution rule,

We'd get a $(\neg a \vee b)$

Commutative

$$\equiv (\neg a \vee b)$$

Make the expression look exactly like the law (more on this later)

Our First Proof

$$\begin{aligned}(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge \text{T}] \\ &\equiv (a \wedge b) \vee [\neg a] \\ &\equiv [\neg a] \vee (a \wedge b) \\ &\equiv (\neg a \vee a) \wedge (\neg a \vee b)\end{aligned}$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at $(\neg a \vee b)$

If we apply the distributive rule,

We'd get a $(\neg a \vee b)$

Distributive $\equiv (\neg a \vee b)$

Creates the $(\neg a \vee b)$ we were hoping for.

Our First Proof

$$\begin{aligned}(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)] \\ &\equiv (a \wedge b) \vee [\neg a \wedge \text{T}] \\ &\equiv (a \wedge b) \vee [\neg a] \\ &\equiv [\neg a] \vee (a \wedge b) \\ &\equiv (\neg a \vee a) \wedge (\neg a \vee b) \\ &\equiv (a \vee \neg a) \wedge (\neg a \vee b) \\ &\equiv \text{T} \wedge (\neg a \vee b)\end{aligned}$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at $(\neg a \vee b)$

If we apply the distribution rule,

We'd get a $(\neg a \vee b)$

Commutative

$$\equiv (\neg a \vee b)$$

Make the expression look exactly like the law (more on this later)

Negation

Simplifies the part we want to disappear.

Simplify $\top \wedge (\neg a \vee b)$ to $(\neg a \vee b)$

For every propositions a, b, r the following hold:

- **Identity**
 - $a \wedge \top \equiv a$
 - $a \vee \text{F} \equiv a$
- **Domination**
 - $a \wedge \text{F} \equiv \text{F}$
 - $a \vee \top \equiv \top$
- **Idempotent**
 - $a \vee a \equiv a$
 - $a \wedge a \equiv a$
- **Commutative**
 - $a \wedge b \equiv b \wedge a$
 - $a \vee b \equiv b \vee a$
- **Associative**
 - $(a \vee b) \vee r \equiv a \vee (b \vee r)$
 - $(a \wedge b) \wedge r \equiv a \wedge (b \wedge r)$
- **Distributive**
 - $a \wedge (b \vee r) \equiv (a \wedge b) \vee (a \wedge r)$
 - $a \vee (b \wedge r) \equiv (a \vee b) \wedge (a \vee r)$
- **Absorption**
 - $a \vee (a \wedge b) \equiv a$
 - $a \wedge (a \vee b) \equiv a$
- **Negation**
 - $a \vee \neg a \equiv \top$
 - $a \wedge \neg a \equiv \text{F}$

Properties of Logical Connectives

We will always give you this list!

For every propositions a, b, r the following hold:

- **Identity**
 - $a \wedge \text{T} \equiv a$
 - $a \vee \text{F} \equiv a$
- **Domination**
 - $a \wedge \text{F} \equiv \text{F}$
 - $a \vee \text{T} \equiv \text{T}$
- **Idempotent**
 - $a \vee a \equiv a$
 - $a \wedge a \equiv a$
- **Commutative**
 - $a \wedge b \equiv b \wedge a$
 - $a \vee b \equiv b \vee a$
- **Associative**
 - $(a \vee b) \vee r \equiv a \vee (b \vee r)$
 - $(a \wedge b) \wedge r \equiv a \wedge (b \wedge r)$
- **Distributive**
 - $a \wedge (b \vee r) \equiv (a \wedge b) \vee (a \wedge r)$
 - $a \vee (b \wedge r) \equiv (a \vee b) \wedge (a \vee r)$
- **Absorption**
 - $a \vee (a \wedge b) \equiv a$
 - $a \wedge (a \vee b) \equiv a$
- **Negation**
 - $a \vee \neg a \equiv \text{T}$
 - $a \wedge \neg a \equiv \text{F}$

Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)]$$

$$\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)]$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at $(\neg a \vee b)$

$$\equiv (a \wedge b) \vee [\neg a \wedge \text{T}]$$

$$\equiv (a \wedge b) \vee [\neg a]$$

$$\equiv [\neg a] \vee (a \wedge b)$$

$$\equiv (\neg a \vee a) \wedge (\neg a \vee b)$$

If we apply the distribution rule,

We'd get a $(\neg a \vee b)$

$$\equiv (a \vee \neg a) \wedge (\neg a \vee b)$$

$$\equiv \text{T} \wedge (\neg a \vee b)$$

$$\equiv (\neg a \vee b) \wedge \text{T}$$

Commutative followed by **Domination** $\equiv (\neg a \vee b)$

Look exactly like the law, then apply it.

We're done!!!

Commutativity

We had the expression $(a \wedge b) \vee [\neg a]$

But before we applied the distributive law, we switched the order...why?

The law says $a \vee (b \wedge r) \equiv (a \vee b) \wedge (a \vee r)$

not $(b \wedge r) \vee a \equiv (b \vee a) \wedge (r \vee a)$

So **technically** we needed to commute first.

Eventually (in about 2 weeks) we'll skip this step. For now, we're doing two separate steps.

Remember this is the "training wheel" stage. The point is to be careful.

More on Our First Proof

We now have an ironclad guarantee that

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv (\neg a \vee b)$$

Hooray! But we could have just made a truth-table. Why a proof?

Here's one reason.

Proofs don't *just* give us an ironclad guarantee. They're also an explanation of *why* the claim is true.

The key insight to our simplification was “the last two pieces were the vacuous truth parts – the parts where *p* was false”

That's in there, *in the proof*.

Our First Proof

$$\begin{aligned} (a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) &\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)] && \text{Associative} \\ &\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)] && \text{Distributive} \\ &\equiv (a \wedge b) \vee [\neg a \wedge \text{T}] && \text{Negation} \\ &\equiv (a \wedge b) \vee [\neg a] && \text{Domination} \\ &\equiv [\neg a] \vee (a \wedge b) && \text{Commutative} \\ &\equiv (\neg a \vee a) \wedge (\neg a \vee b) && \text{Distributive} \\ &\equiv (a \vee \neg a) \wedge (\neg a \vee b) && \text{Commutative} \\ &\equiv \text{T} \wedge (\neg a \vee b) && \text{Negation} \\ &\equiv (\neg a \vee b) \wedge \text{T} && \text{Commutative} \\ &\equiv (\neg a \vee b) && \text{Domination} \end{aligned}$$

The last two terms are "vacuous truth" – they simplify to $\neg a$

p no longer matters in $a \wedge b$ if $\neg a$ automatically makes the expression true.

More on Our First Proof

With practice (and quite a bit of squinting) you can see not just the ironclad guarantee, but also the reason why something is true.

That's not easy with a truth table.

Proofs can also communicate intuition about why a statement is true. We'll practice extracting intuition from proofs more this quarter.

Converse, Contrapositive

Implication:

$$a \rightarrow b$$

If it's raining, then I have my umbrella.

Converse:

$$b \rightarrow a$$

If I have my umbrella, then it is raining.

Contrapositive:

$$\neg b \rightarrow \neg a$$

If I don't have my umbrella, then it is not raining.

Inverse:

$$\neg a \rightarrow \neg b$$

If it is not raining, then I don't have my umbrella.

How do these relate to each other?

a	b	$a \rightarrow b$	$b \rightarrow a$	$\neg a$	$\neg b$	$\neg a \rightarrow \neg b$	$\neg b \rightarrow \neg a$
T	T						
T	F						
F	T						
F	F						

Converse, Contrapositive

Implication:

$$a \rightarrow b$$

Converse:

$$b \rightarrow a$$

Contrapositive:

$$\neg b \rightarrow \neg a$$

Inverse:

$$\neg a \rightarrow \neg b$$

An **implication** and its **contrapositive**
have the same truth value!

a	b	$a \rightarrow b$	$b \rightarrow a$	$\neg a$	$\neg b$	$\neg a \rightarrow \neg b$	$\neg b \rightarrow \neg a$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Contrapositive

Activity

We showed $a \rightarrow b \equiv \neg b \rightarrow \neg a$ with a truth table. Let's do a proof.

Try this one on your own. Remember

1. Know what you're trying to show.
2. Stay on target – take steps to get closer to your goal.

Hint: think about your tools. There are lots of rules with AND/OR/NOT, but very few with implications...

Contrapositive

$a \rightarrow b \equiv \neg a \vee b$	Law of Implication
$\equiv b \vee \neg a$	Commutativity
$\equiv \neg \neg b \vee \neg a$	Double Negation
$\equiv \neg b \rightarrow \neg a$	Law of Implication

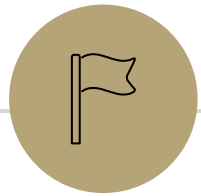
All of our rules deal with ORs and ANDs, let's switch the implication to just use AND/NOT/OR.

And do the same with our target

It's ok to work from both ends. In fact it's a very common strategy!

Now how do we get the top to look like the bottom?

Just a few more rules and we're done!



Digital Logic



Digital Circuits

Computing With Logic

T corresponds to **1** or “high” voltage

F corresponds to **0** or “low” voltage

Gates

Take inputs and produce outputs (functions)

Several kinds of gates

Correspond to propositional connectives (most of them)

And Gate

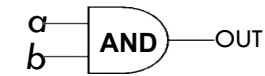
AND Connective

vs.

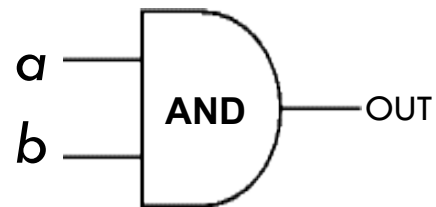
AND Gate

$a \wedge b$

a	b	$a \wedge b$
T	T	T
T	F	F
F	T	F
F	F	F



a	b	OUT
1	1	1
1	0	0
0	1	0
0	0	0



“block looks like D of AND”

Or Gate

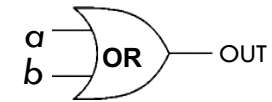
OR Connective

vs.

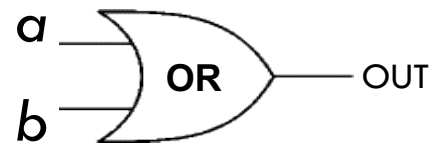
OR Gate

$a \vee b$

a	b	$a \vee b$
T	T	T
T	F	T
F	T	T
F	F	F



a	b	OUT
1	1	1
1	0	1
0	1	1
0	0	0



“arrowhead block looks like V”

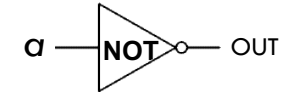
Not Gates

NOT Connective

vs.

NOT Gate

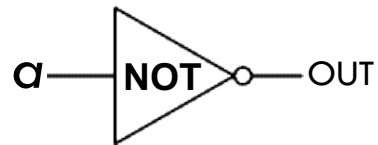
$\neg a$



Also called
inverter

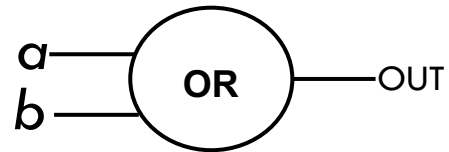
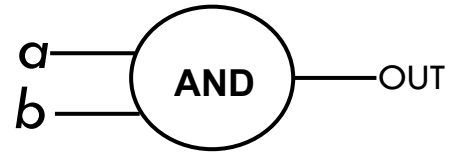
a	$\neg a$
T	F
F	T

a	OUT
1	0
0	1

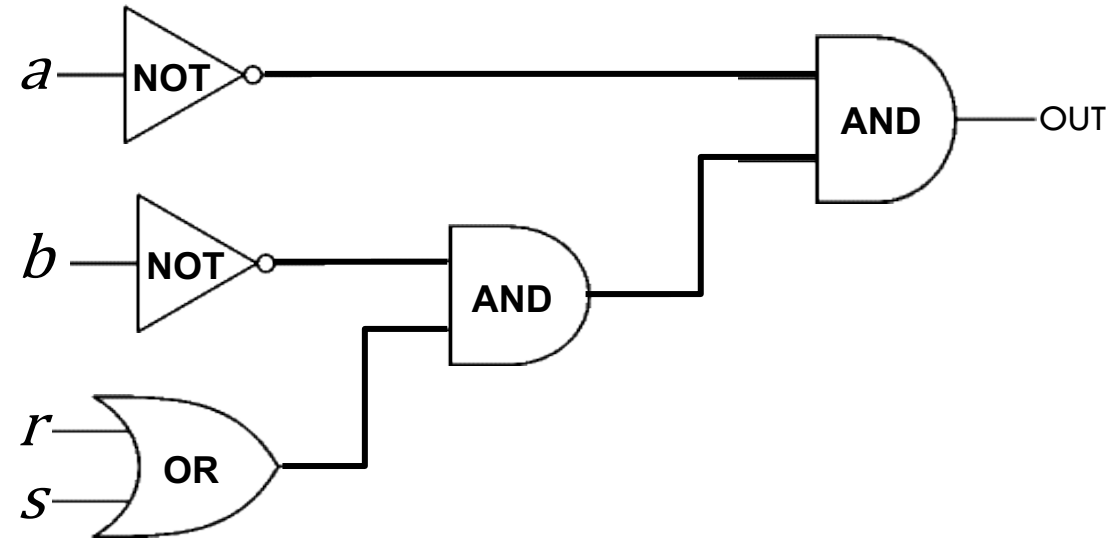


Blobs are Okay!

You may write gates using blobs instead of shapes!

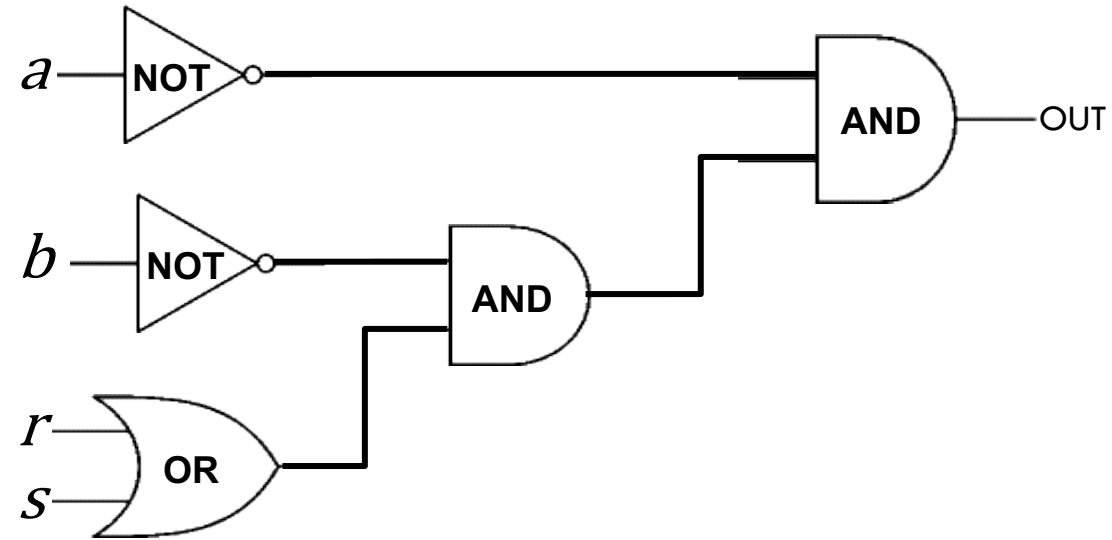


Combinational Logic Circuits



Values get sent along wires connecting gates

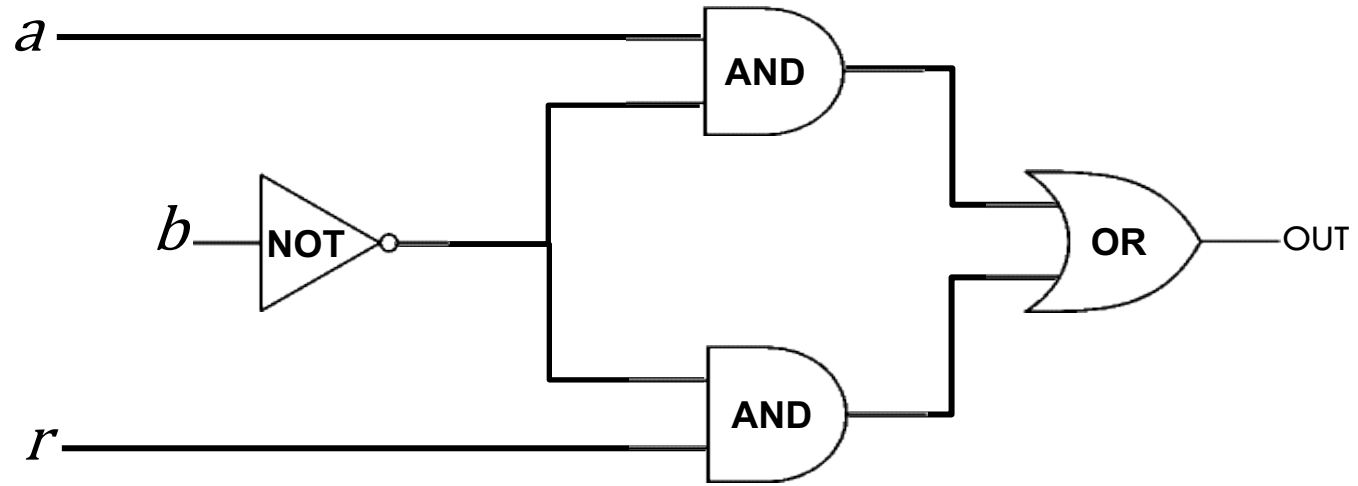
Combinational Logic Circuits



Values get sent along wires connecting gates

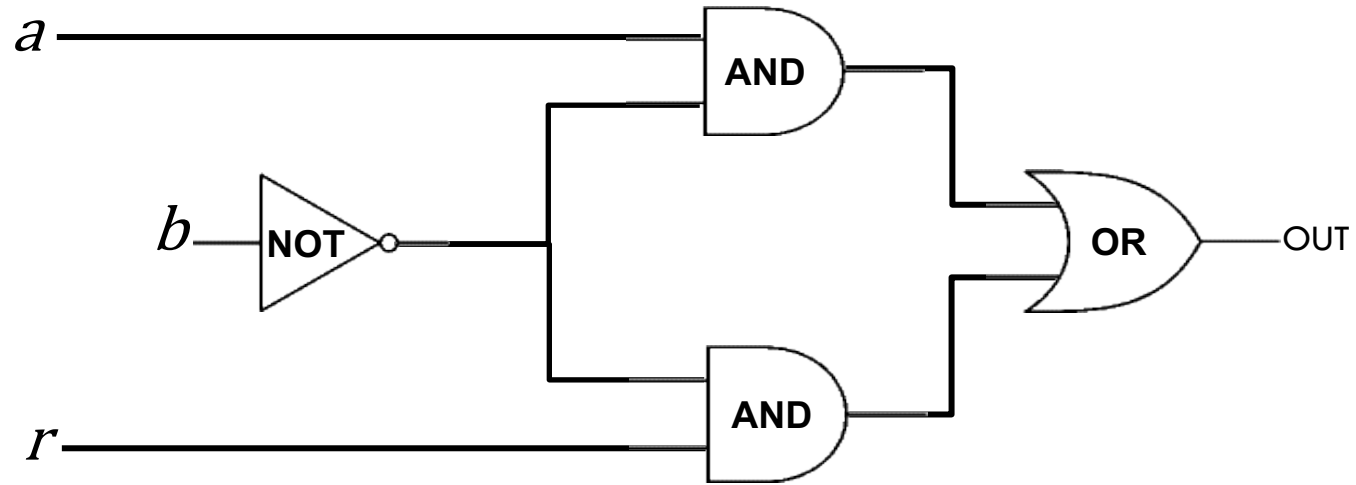
$$\neg p \wedge (\neg q \wedge (r \vee s))$$

Combinational Logic Circuits



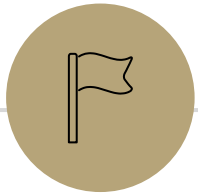
Wires can send one value to multiple gates!

Combinational Logic Circuits



Wires can send one value to multiple gates!

$$(p \wedge \neg q) \vee (\neg q \wedge r)$$



More Vocabulary

Vocabulary!

A proposition is a....

Tautology if it is always true.

Contradiction if it is always false.

Contingency if it can be both true and false.

$a \vee \neg a$

Tautology

If a is true, $a \vee \neg a$ is true; if a is false, $a \vee \neg a$ is true.

$a \oplus a$

Contradiction

If a is true, $a \oplus a$ is false; if a is false, $a \oplus a$ is false.

$(a \rightarrow b) \wedge a$

Contingency If a is true and b is true, $(a \rightarrow b) \wedge a$ is true;
If a is true and b is false, $(a \rightarrow b) \wedge a$ is false.

Another Proof

Let's prove that $(a \wedge b) \rightarrow (b \vee a)$ is a tautology.

Alright, what are we trying to show?

Another Proof

$$\begin{aligned}(a \wedge b) \rightarrow (b \vee a) &\equiv \neg(a \wedge b) \vee (b \vee a) \\ &\equiv (\neg a \vee \neg b) \vee (b \vee a) \\ &\equiv \neg a \vee (\neg b \vee (b \vee a)) \\ &\equiv \neg a \vee ((\neg b \vee b) \vee a) \\ &\equiv \neg a \vee ((b \vee \neg b) \vee a) \\ &\equiv \neg a \vee (T \vee a) \\ &\equiv \neg a \vee (a \vee T) \\ &\equiv \neg a \vee a \\ &\equiv a \vee \neg a \\ &\equiv T\end{aligned}$$

Proof-writing tip:

Take a step back.

Pause and carefully look at what you have. You might see where to go next...

Law of Implication

DeMorgan's Law

It's easier if everything is AND/OR/NOT

Associative (twice)

Gets rid of some parentheses/just a gut feeling

Commutative, Negation

Put $b, \neg b$ next to each other.

Commutative, Domination

Simplify out the $b, \neg b$

Commutative, Negation

Simplify out the $1, \neg 1$.

We're done!

Another Proof

$(a \wedge b) \rightarrow (b \vee a)$	$\equiv \neg(a \wedge b) \vee (b \vee a)$	Law of implication
	$\equiv (\neg a \vee \neg b) \vee (b \vee a)$	DeMorgan's Law
	$\equiv \neg a \vee (\neg b \vee (b \vee a))$	Associative
	$\equiv \neg a \vee ((\neg b \vee b) \vee a)$	Associative
	$\equiv \neg a \vee ((b \vee \neg b) \vee a)$	Commutative
	$\equiv \neg a \vee (T \vee a)$	Negation
	$\equiv \neg a \vee (a \vee T)$	Commutative
	$\equiv \neg a \vee a$	Domination
	$\equiv a \vee \neg a$	Commutative
	$\equiv T$	Negation

Computing Equivalence

Given two propositions, can we write an algorithm to determine if they are equivalent?

What is the runtime of our algorithm?

Computing Equivalence

Given two propositions, can we write an algorithm to determine if they are equivalent?

Yes! Generate the truth tables for both propositions and check if they are the same for every entry.

What is the runtime of our algorithm?

Every atomic proposition has two possibilities (T, F). If there are n atomic propositions, there are 2^n rows in the truth table.