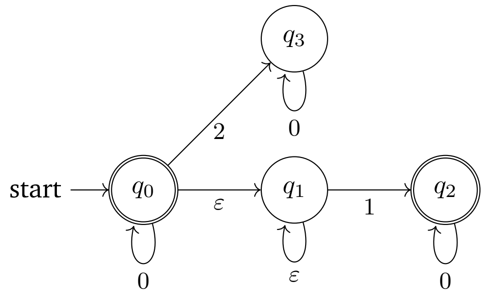
Section 09: NFAs and Minimization

1. NFAs

(a) What language does the following NFA accept?

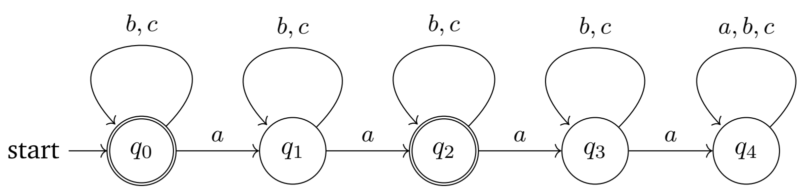


(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

2. DFAs & Minimization

(a) Convert the NFA from 1a to a DFA, then minimize it.

(b) Minimize the following DFA:



3. Review: Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- `soy`(x) is true iff x contains soy milk.
- `whole`(x) is true iff x contains whole milk.
- `sugar`(x) is true iff x contains sugar
- `decaf`(x) is true iff x is not caffeinated.
- `vegan`(x) is true iff x is vegan.
- `RobbieLikes`(x) is true iff Robbie likes the drink x.

Translate each of the following statements into predicate logic. You may use quantifiers, the predicates above, and usual math connectors like $=$ and $\neq$.

(a) Coffee drinks with whole milk are not vegan.

(b) Robbie only likes one coffee drink, and that drink is not vegan.

(c) There is a drink that has both sugar and soy milk.

Translate the following symbolic logic statement into a (natural) English sentence. Take advantage of domain restriction.

$$\forall x([\texttt{decaf}(x) \wedge \texttt{RobbieLikes}(x)] \rightarrow \texttt{sugar}(x))$$

4. Review: Number Theory

Let p be a prime number at least 3, and let x be an integer such that $x^2\%p = 1$.

(a) Show that if an integer y satisfies $y \equiv 1 \pmod p$, then $y^2 \equiv 1 \pmod p$. (this proof will be short!)
(Try to do this without using the theorem ”Raising Congruences To A Power”)

(b) Repeat part (a), but don't use any theorems from the Number Theory Reference Sheet. That is, show the claim directly from the definitions.

(c) From part (a), we can see that $x\%p$ can equal 1. Show that for any integer x, if $x^2 \equiv 1 \pmod p$, then $x \equiv 1 \pmod p$ or $x \equiv -1 \pmod p$. That is, show that the only value $x\%p$ can take other than 1 is $p - 1$.
Hint: Suppose you have an x such that $x^2 \equiv 1 \pmod p$ and use the fact that $x^2 - 1 = (x - 1)(x + 1)$
Hint: You may the following theorem without proof: if p is prime and $p \mid (ab)$ then $p \mid a$ or $p \mid b$.