Section 08: Relations, DFAs

1. Relations

(a) Draw the transitive-reflexive closure of $\{(1,2),(2,3),(3,4)\}$.

(b) Suppose that R is reflexive. Prove that $R \subseteq R^2$.

(c) Consider the relation $R = \{(x,y) \ : \ x = y+1\}$ on $\mathbb{N}$. Is R reflexive? Transitive? Symmetric? Anti-symmetric?

(d) Consider the relation $S = \{(x,y) \ : \ x^2 = y^2\}$ on $\mathbb{R}$. Prove that S is reflexive, transitive, and symmetric.

2. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0,1,2,3\}$.

(a) All binary strings.

(b) All strings whose digits sum to an even number.

(c) All strings whose digits sum to an odd number.

3. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0,1\}$.

(a) All strings which do not contain the substring 101.

(b) All strings containing at least two 0's and at most one 1.

(c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

4. Relations and Strings

Let $\Sigma = \{0,1\}$ and define the relation $\diamond$ on Σ^* by $x \diamond y$ if and only if the length of xy is even. (Here $x \diamond y$ is another way of writing $(x,y) \in \diamond$.) Prove that $\diamond$ is reflexive, symmetric, and transitive.