# Section 02: Review - Proofs, Digital Logic, Boolean Algebra, Predicate Logic

## 1. Laws Relating to Equivalence

(Refer to the Logical Equivalences reference sheet for a full list.)

- **De Morgan's Laws:**

$$\neg(p \land q) \equiv \neg p \lor \neg q$$
$$\neg(p \lor q) \equiv \neg p \land \neg q$$

- **Law of Implication:**

$$p \to q \equiv \neg p \lor q$$

- **Contrapositive:**

$$p \to q \equiv \neg q \to \neg p$$

- **Biconditional**

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$

- **Double Negation:**

$$p \equiv \neg\neg p$$

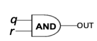
## 2. Digital Circuits

### And Gate

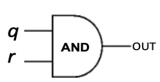
**AND Connective** vs. **AND Gate**

$q \land r$

| $q$ | $r$ | $q \land r$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |




| $q$ | $r$ | OUT |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |



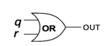

"block looks like D of AND"

### Or Gate

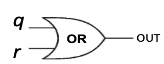
**OR Connective** vs. **OR Gate**

$q \lor r$

| $q$ | $r$ | $q \lor r$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |




| $q$ | $r$ | OUT |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |



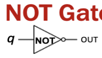

"arrowhead block looks like V"

### Not Gates

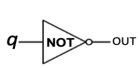
**NOT Connective** vs. **NOT Gate** (Also called *inverter*)

$\neg q$

| $q$ | $\neg q$ |
|---|---|
| T | F |
| F | T |




| $q$ | OUT |
|---|---|
| 1 | 0 |
| 0 | 1 |



## 3. Boolean Algebra

- **Canonical form:** Standard form for a boolean expression
- **Sum of Products Canonical Form (AKA Disjunctive Normal Form (DNF) or Minterm Expansion):**
  - Identify the rows with output True in the truth table.
  - Take the product terms for these rows by ANDing the literals (input combination).
  - OR the identified product terms. This is the DNF canonical form expression.
- **Product of Sums Canonical Form (AKA Conjunctive Normal Form (CNF) or Maxterm Expansion):**
  - Identify the rows with output False in the truth table.
  - Take the sum terms for these rows by ORing the negated literals (input combination).
  - AND the identified sum terms. This is the CNF canonical form expression.

## 4. Predicate Logic

- **Predicate:** A function that returns a truth value (True or False). Can have varying numbers of arguments and input types.
- **Domain of discourse:** Non-empty set of objects that a predicate is defined over
- **Universal quantifier:** Represented by $\forall$ (read as "for all"). Often translated as "for all", "for each", or "for every".
- **Existential quantifier:** Represented by $\exists$ (read as "there exists"). Often translated as "there exists", "there is", or "for some".
- **De Morgan's Laws for Quantifiers:**

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg\exists x P(x) \equiv \forall x \neg P(x)$$

## 5. Guidelines for Translating Predicate Logic

- When there is no leading quantification, it generally means "for all".
- Some means "there exists"
- **Domain Restriction**
  - When restricting to a smaller domain in a "for all", we use implication.
  - When restricting to a smaller domain in an "exists", we use and.