Section 01: Solutions

1. Warm-Up

Translate the English sentences below into symbolic logic.

(a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Solution:

Since we're in "if...then..." form, the sentence is an implication.
a: I am lifting weights
b: I do a warm-up exercise

$$a \to b$$

(b) If I am cold and going to bed or I am two-years old, then I carry a blanket. **Solution:**

a: I am cold
b: I am going to bed
c: I am two-years old
d: I carry a blanket

$$[(a \wedge b) \vee c] \to d$$

How did we know the translation wasn't $[a \wedge (b \vee c)] \to d$? Two hints were available: first, omitted words ("going to bed" instead of "I am going to bed" indicates b should be closer to the "and" than the "or"), second the interpretation of the sentence – two-year olds more commonly carry blankets during the day than warm adults.

2. Truth Tables

Write a truth table for each of the following:

(a) $(r \oplus q) \vee (r \oplus \neg q)$ **Solution:**

r	q	$r \oplus q$	$r \oplus \neg q$	$(r \oplus q) \vee (r \oplus \neg q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

(b) $(r \vee q) \to (r \oplus q)$ **Solution:**

r	q	$r \vee q$	$r \oplus q$	$(r \vee q) \rightarrow (r \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

(c) $p \leftrightarrow \neg p$ **Solution:**

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

3. If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

(a) whenever I walk my dog, I make new friends. **Solution:**

p: I walk my dog
r: I make new friends

$$p \rightarrow r$$

The promise is that we will definitely make new friends on the condition of walking our dog.

(b) I will drink coffee, if Starbucks is open or my coffeemaker works.

Solution:

a: I will drink coffee
b: Starbucks is open
c: my coffeemaker works

$$(b \vee c) \rightarrow a$$

(c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote. **Solution:**

a: One is a U.S. citizen
b: One is over 18
c: One is eligible to vote

$$(a \wedge b) \rightarrow c$$

The original sentence omits a subject. We introduced a dummy subject "one" to the propositions, you

might have said “someone” or “a person” instead (among other options).

(d) I can go home only if I have finished my homework. **Solution:**

p: I can go home.
r: I have finished my homework.

$$p \to r$$

The promise here is that if I can go home then I must have finished my homework. It can sometimes help to imagine when the sentence is broken. Is it broken if my homework is finished, but I cannot go home? No, perhaps I also have to say bye to my friends before I leave. But if I can go home with unfinished homework, then the promise is broken.

“Only if” is one of the more confusing arrangements – the consequence (“the then part”) is adjacent to the “only if.”

(e) Having an internet connection is necessary to log onto zoom. **Solution:**

p: One has an internet connection
r: One can log onto zoom

$$r \to p$$

The internet connection is not enough (what if you don’t have the meeting link?) but certainly if you are in the meeting then you have a connection.

4. I can rewrite these formulas in English, only if...

Given propositions and a logical formula, write **two** potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

(a) p: The sun is out
r: We have class outside

$$p \to r$$

Solution:

If the sun is out, then we have class outside.
Whenever the sun is out, we have class outside.

(b) a: the book has been out for a week.
b: I don’t have homework.
c: I have finished reading the book.

$$(a \wedge b) \to c$$

Solution:

I have finished reading the book, if it has been out for a week and I don’t have homework.
The book being out for a week and me not having homework is sufficient for me to have finished reading the book.

(c) p: I have read the manual
r: I operate the machine

$$r \rightarrow p$$

Solution:

I operate the machine only if I have read the manual.
Operating the machine implies that I have read the manual.

5. Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

(a) I will remember to send you the address only if you send me an e-mail message. **Solution:**

p : I will remember to send you the address
r : You send me an e-mail message

$$\boxed{p \rightarrow r}$$

(b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area. **Solution:**

a : Berries are ripe along the trail
b : Hiking is safe
c : Grizzly bears have not been seen in the area

$$\boxed{a \rightarrow (b \leftrightarrow c)}$$

(c) Unless I am trying to type something, my cat is either eating or sleeping. **Solution:**

a : My cat is eating
b : My cat is sleeping
c : I'm trying to type

$$\boxed{\neg c \rightarrow (a \oplus b)}$$

6. Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

(a) Define propositional variables and translate the sentence into an expression in logical notation. **Solution:**

p : I am drinking tea
r : I am eating a cookie

$$\boxed{(p \to r) \vee (r \to p)}$$

(b) Fill out a truth table for your expression. **Solution:**

p	r	$(p \to r)$	$(r \to p)$	$(p \to r) \vee (r \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

7. ??clusive Or

Exclusive or ($\oplus$) and inclusive or ($\vee$) both can be translated as "or" in English. For each of the following ambiguous phrases, decide which type of "or" is likely meant and why.

(a) Experience with C or Java is required. **Solution:**

Inclusive or. Experience with both is usually not a bad thing.

(b) Lunch includes soup or salad. **Solution:**

Exclusive or. Most restaurants charge you more for both.

(c) Publish or perish. **Solution:**

This phrase is a common one among researchers – it means publish papers or your career will perish. Exclusive or is meant; i.e. if you do indeed publish you should avoid the loss of your career.

(d) To enter the country, you need a passport or voter registration card. **Solution:**

Inclusive or – if you have both, they won't kick you out!

8. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a) $p \to r$ vs. $r \to p$ **Solution:**

When $p = \mathsf{T}$ and $r = \mathsf{F}$, then $p \to r \equiv \mathsf{F}$, but $r \to p \equiv \mathsf{T}$.

(b) $a \to (b \wedge c)$ vs. $(a \to b) \wedge c$ **Solution:**

When $a = \mathsf{F}$ and $c = \mathsf{F}$, then $a \to (b \wedge c) \equiv \mathsf{T}$ (by vacuous truth), but $(a \to b) \wedge c \equiv \mathsf{F}$ (because c is false).

9. They mean the same thing

In the activity from lecture 2, we showed the following.

$$\neg(q \to r) \equiv \neg(\neg q \vee r)$$

Use the elementary equivalences presented at the end of lecture 2 to argue that the following pairs are equivalent.

$$\neg(\neg q \vee r) \equiv \neg(\neg q) \wedge \neg r \quad (1)$$
$$\neg(\neg q) \wedge \neg r \equiv q \wedge \neg r \quad (2)$$
$$q \wedge \neg r \equiv \neg r \wedge q \quad (3)$$

Your friend says this means that $\neg(q \to r) \equiv \neg r \wedge q$. Is that true?

Solution:

$$\begin{aligned} \neg(q \to r) &\equiv \neg(\neg q \vee r) && \text{(Activity in Lecture 2)} \\ &\equiv \neg(\neg q) \wedge \neg r && \text{(De Morgan)} \\ &\equiv q \wedge \neg r && \text{(Double negation)} \\ &\equiv \neg r \wedge q && \text{(Commutativity)} \end{aligned}$$

For any statements A, B, and C, if A and B agree on all possible truth assignments and B and C do too, then A and C agree on all possible truth assignments, so the above chain of equivalences shows that $\neg(q \to r) \equiv \neg r \wedge q$.

10. Equivalent Translations

Prove that the following English statements are equivalent.
(i) Unless it isn't raining or I don't have an umbrella, I buy a book.
(ii) It isn't raining or I don't have an umbrella or I buy a book. **Solution:**

a : It is raining.

b : I have an umbrella.

c : I buy a book.

When we say unless a, b, this suggests that as long as a is not true, b will be true. Then, we can rewrite (i) as follows:

$$\neg(\neg a \vee \neg b) \to c$$

With the same propositional variables, we can rewrite (ii) as:

$$\neg a \vee \neg b \vee c$$

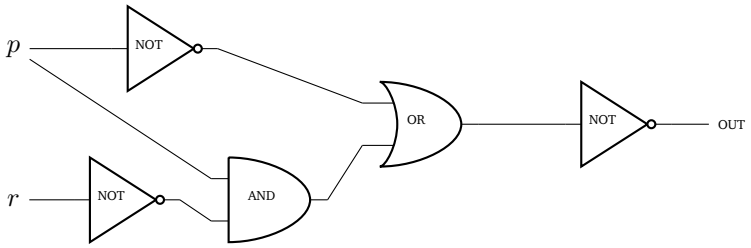
If these two compound propositions are equivalent, then the English statements are equivalent. Starting with the left-hand side

$$\begin{aligned} \neg(\neg a \vee \neg b) \to c &\equiv (\neg\neg a \wedge \neg\neg b) \to c && \text{[DeMorgan's Laws]} \\ &\equiv (a \wedge b) \to c && \text{[Double Negation]} \\ &\equiv \neg(a \wedge b) \vee c && \text{[Law of Implication]} \\ &\equiv (\neg a \vee \neg b) \vee c && \text{[DeMorgan's Laws]} \end{aligned}$$

Therefore, we've shown that the two English statements are equivalent.

11. Circuitous

Translate the following circuit into a logical expression.



Solution:

$\neg(\neg p \vee (p \wedge \neg r))$