

Section 01: Review: Propositional Logic, Equivalences, Proofs

1. Propositional Logic

- **Proposition:** A statement that has a truth value (i.e. is true or false) and is “well-formed”.
- Propositional variables use lower-case letters (a,b,c,..) to represent propositions.

1.1. Logical Connectives

- Order of operations:
 - (a) Parentheses
 - (b) Negation
 - (c) And
 - (d) Or
 - (e) Implication
 - (f) Biconditional

1.1.1. Conjunction (And)

a	b	$a \wedge b$
T	T	T
T	F	F
F	T	F
F	F	F

1.1.2. Disjunction (Or)

a	b	$a \vee b$
T	T	T
T	F	T
F	T	T
F	F	F

1.1.3. Exclusive Or

a	b	$a \oplus b$
T	T	F
T	F	T
F	T	T
F	F	F

1.1.4. Negation (Not)

a	$\neg a$
T	F
F	T

1.1.5. Implication

a	b	$a \rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

Possible English translations:

- a implies b
- whenever a is true b must be true
- if a then b
- b if a
- a is sufficient for b
- a only if b
- b is necessary for a

1.1.6. Biconditional

a	b	$a \leftrightarrow b$
T	T	T
T	F	F
F	T	F
F	F	T

- Equivalent to $a \rightarrow b \wedge b \rightarrow a$
- Equivalent to $\neg a \leftrightarrow \neg b$

Possible English translations:

- a if and only if (iff) b
- a is equivalent to b
- a implies b and b implies a
- a is necessary and sufficient for b

2. Logical Equivalences

- **Equivalence:** When two propositions always have the same truth value.
- $(A \leftrightarrow B) \equiv T$ and $A \equiv B$ mean the same thing
- **Tautology:** when a statement (compound proposition) is always true
- **Contradiction:** when a statement (compound proposition) is always false
- **Contingency:** when a statement (compound proposition) can be either true or false