Homework 2: Propositional Logic and Predicate Logic

Due date: Wednesday, April 14 at 11:00 PM (Seattle time, i.e. GMT-7)
If you work with others (and you should!), remember to follow the collaboration policy.
In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting.

Be sure to read the grading guidelines for more information on what we're looking for.

While working on the assignment, please (roughly) keep track of the time you spent on each problem. This is so that you have an idea of how to answer the feedback questions!

1. Collaborators

List any collaborators (i.e., other students) you worked with and which problems you worked on together, or state that you worked alone.

2. Think Contrapositive Be Contrapositive [14 points]

(a) If I go to the store and I cook for myself, then I will make soup.

  (i) convert this sentence to propositional logic (as on homework 1, ensure you're giving variables to atomic propositions, not compound ones). [2 points]

  (ii) take the contrapositive symbolically, and simplify so that $\neg$ signs are next to atomic propositions (i.e. only single variables). [2 points]

  (iii) translate the contrapositive back to English. [3 points]

(b) In order to rent a car, it is necessary to have a driver's license.

Repeat steps (i)-(iii) from (a) for this sentence.

3. Some Symbols [10 points]

Prove that $(a \to b) \vee (c \to b) \equiv (a \wedge c) \to b$

For this problem, you need to write a symbolic proof using a chain of equivalences. To construct this proof, you should use propositional logic notation and rules. You should also follow the symbolic proof guidelines.

Our proof has three "intermediate goals":

(a) Convert to only ands/ors/nots with only atomic propositions negated

(b) Rearrange to eliminate the "extra" b

(c) Rearrange to final expression

Your proof is allowed to go differently (we will accept any correct, properly formatted proof), but our intermediate goals may help you if you are stuck.

4. Two of a kind [12 points]

Show that the propositional logic formula $\neg(\neg(a \wedge (a \vee b)) \vee (a \wedge (b \vee (a \vee \neg b))))$ is a contradiction. Do so using a chain of equivalences. Please follow the symbolic proof guidelines.

5. The New Normal Form [10 points]

Consider the following function $C(x, y, z)$:

x	y	z	$C(x,y,z)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

(a) Express C in Disjunctive Normal Form. [5 points]

(b) Express C in Conjunctive Normal Form. [5 points]

6. A tale of two $\exists$ [12 points]

Consider the following two expressions:

$$\exists x(\mathsf{P}(x) \wedge \mathsf{Q}(x)) \qquad \exists x\, \mathsf{P}(x) \wedge \exists x\, \mathsf{Q}(x)$$

(a) Give a domain of discourse and definitions of P and Q such that these expressions are **not** equivalent. Explain why your examples work (1-2 sentences). [6 points]

(b) Give a domain of discourse and definitions of P and Q such that these expressions **are** equivalent. Explain why your examples work (1-2 sentences). [6 points]

(c) **Extra Credit:** There is a logical relationship between these two expressions (one that is true for all domains and all predicates P, Q). By "logical relationship" we mean there is a logical connective that can join the two expressions together into a single true expression. What is that combined expression? Very briefly summarize why the relationship is true (1-2 sentences).

7. Inside Baseball

In the beforetimes, you went to a UW baseball game with two friends on "Bark at the Park" day. Husky Baseball Stadium rules do not allow for non-human mammals to attend, except as follows: (1) Dubs is allowed at every game (2) if it is "Bark at the Park" day, everyone can bring their pet dogs. You let your domain of discourse be all mammals at the game.

The predicates Dog, Dubs, Human are true if and only if the input is a dog, Dubs, or a human respectively. UW is facing the Oregon State Beavers. The predicate HuskyFan(x) means "x is a Husky fan" and similarly for BeaverFan. Finally HavingFun is true if and only if the input mammal is having fun right now.

7.1. Strike One [16 points]

One of your friends hands you the following observations; translate them into English. Your translations should take advantage of "restricting the domain" to make more natural translations when possible, but you should not otherwise simplify the expression before translating.

(a) $\forall x\,(\texttt{Dog}(x) \to [\texttt{Dubs}(x) \vee \texttt{BeaverFan}(x)])$

(b) $\exists x\,(\texttt{HuskyFan}(x) \wedge \texttt{Human}(x) \wedge \neg\,\texttt{HavingFun}(x))$

(c) $\forall x(\texttt{BeaverFan}(x) \to \neg\,\texttt{HavingFun}(x)) \wedge \forall x(\texttt{HuskyFan}(x) \vee \texttt{Dubs}(x) \to \texttt{HavingFun}(x))$

(d) $\neg\exists x\,(\texttt{Dog}(x) \wedge \texttt{HavingFun}(x) \wedge \texttt{BeaverFan}(x))$

7.2. Strike Two [8 points]

You realize that the first two sentences above are false.

(a) State the negation of (a) in English. You should simplify the negation so that the English sentence is natural.

(b) Repeat the directions above for sentence (b).

8. There is an implication [8 points]

Implications are uncommon under existential quantifiers. Consider this expression (which we'll call "the original expression"): $\exists x(P(x) \to Q(x))$

(a) Suppose that $P(x)$ is not always true (i.e. there is an element in the domain for which $P(x)$ is false). Explain why the original expression is true in this case. (1-2 sentences should suffice. If you prefer, you may give a formal proof instead). [4 points]

(b) Suppose that $P(x)$ is always true (i.e. $\forall x P(x)$). There is a simpler statement which conveys the meaning of the original expression (i.e. is equivalent to it for all domains and predicates. By simpler, we mean "uses fewer symbols"). Give that expression, and briefly (1-2 sentences) explain why it works. [4 points]

(c) Ponder, based on the last two parts, why it's very uncommon to write the original expression. You do not have to write anything for this part, simply ponder. [0 points]

9. Extra credit: At Last

In this problem, you will design a circuit with a minimal number of gates that takes a pair of four-bit integers $(x_3x_2x_1x_0)_2$ and $(y_3y_2y_1y_0)_2$ and returns a single bit indicating whether $x_3x_2x_1x_0 < y_3y_2y_1y_0$. See the following table for some examples.

$x_3x_2x_1x_0$	$y_3y_2y_1y_0$	$x_3x_2x_1x_0 < y_3y_2y_1y_0$
0101	1011	1
1100	0111	0
1101	1101	0

Design such a circuit using at most 10 AND, OR, and XOR gates. You can use an arbitrary number of NOT gates. The AND and OR gates can have multiple inputs. The smaller the solution, the better.

10. Feedback

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- How many hours did you spend working on this assignment?

- Which problem did you spend the most time on?
- Any other feedback for us?