Homework 1: Propositional Logic

Changelog: This is version 2, updated 4/3. We updated the variable names in problem 6a to q and r.

Due date: Wednesday, April 7 at 11:00 PM (Seattle time, i.e. GMT-7)
If you work with others (and you should!), remember to follow the collaboration policy.
In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting.

Be sure to read the grading guidelines for more information on what we're looking for.

1. Translation [16 points]

Translate the English statements into symbolic logic, making the atomic propositions as basic as possible.

(a) To watch Tangled, it suffices to have a DisneyPlus account or a DVD. [4 points]
Note: propositions should be able to have a truth value, so "watch Tangled" is not a proposition, but "One can watch Tangled" is, for example.

(b) The file can be created only if the disk is not full and the user has write permissions. [4 points]

(c) Define a set of *at most three* atomic propositions. Then, use them to translate all of these sentences about why dogs lick you into logic. [8 points]

(i) If the dog licks you, then they show you affection.

(ii) Unless they want attention, the dog does not lick you.

(iii) The dog licks you only if they show you affection or want attention.

2. Trickier translation [10 points]

The following sentences are idiomatic in English...but not very clear logically. Convert each statement into propositional logic, then write an English sentence that has the same meaning, but is clearer logicially.

(a) If you leave a room, you should turn the light off, unless there is someone in the room or you don't see the lightswitch.

(b) I can go home, if the lightrail is running, and if my lightrail pass is active as well.

3. Inequivalence [16 points]

For each part, find a truth assignment (i.e. an assignment of `True` or `False` to $q, r,$ and s) to show the pair of statements are not equivalent. Explain why your assignments work (our explanations are 1-2 sentences).

(a) $(q \to s) \to r$ vs. $q \to (r \to s)$.

(b) $s \vee (s \wedge q)$ vs. $s \vee q$.

(c) $s \to (q \wedge r)$ vs. $(s \to q) \wedge r$

(d) $(s \wedge q) \to r$ vs. $(\neg s \wedge \neg q) \vee r$

4. Compound Proposition [8 points]

Find a compound proposition involving the variables $a, b, c,$ and d that is true precisely when **at least** two of $a, b, c,$ and d are true. Explain why your answer works (1-2 sentences). *Note: By "precisely," we mean also that the proposition should be false whenever the condition is not met.*

5. Would You, Could You, With a Gate? [10 points]

Consider the following gates.

AND Gates, **OR** Gates, and Inverters (**NOT** Gates),

Using only **AND**, **OR**, and **NOT** gates, draw the diagram of a circuit with **three input wires** that computes the function $B(q, r, s)$, defined as follows.

- If q is false or r is false, then $B(q, r, s) = s$.
- Otherwise, $B(q, r, s) = \neg s$.

Your diagram should have one output wire.

Note: For this exercise it might be most time efficient to draw the diagram by hand, then take a picture and include it in your latex file (see here). Also note that you can use each input multiple times, and you can indicate wires crossing without touching with something like this.

6. Proof [20 points]

In Lecture 3 (here) we gave a symbolic proof that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r) \equiv \neg q \vee r$. In this problem we will give another proof.

(a) Our intuition for the proof in class was "the second and third terms of the formula correspond to vacuous truth in $q \to r$." Identify something common to the first two pieces ($q \wedge r$ and $\neg q \wedge r$) but not the third ($\neg q \wedge \neg r$), and describe it.
(Your description should be similar in spirit to the one from class, but you don't need to use fancy vocabulary like "vacuous truth" – our answer here is one sentence) [4 points]

(b) Give another proof of the formula that matches the intuition from part (a) instead of the intuition from class. Read the symbolic proof guidelines before you start. [12 points]
Hint: your proof, if it matches your intuition from (a) will be different from the one from class – at least some of the intermediate expressions will be different.

(c) In class we labeled portions of the proof with high-level descriptions of what they are doing (purple text, left side). Produce similar labels for your proof. Submit your answer in the form "Steps [X] to [Y]: [label]" for each part. [4 points]
Note: The goal here is to give intuition for what is happening at a higher level than individual steps.

7. Extra Credit

Computers have storage spaces called "registers" (they are placed right near the processing unit to hold the values urgently needed for upcoming calculations). A register is a fixed number of bits long (i.e. a fixed number of `T` or `F`). For any two bits a, b we define $\texttt{XNOR}(a, b) := \neg(a \oplus b)$.

Suppose you have two memory registers R_i and R_j. You have only one operation available: XNOR(R_i, R_j) performs XNOR bit-by-bit and **stores the result back in** R_i. By "bit-by-bit" we mean we XNOR the k^{th} bit of R_i with the k^{th} bit of R_j to get the k^{th} bit of the result).

Show that you can swap the contents of R_i and R_j using only XNOR operations and **only** the registers R_i, R_j – you are not allowed any "temporary variables" or other registers. Give both a list of steps and a brief explanation of how your solution works.

8. Feedback

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- How many hours did you spend working on this assignment?
- Which problem did you spend the most time on?
- Any other feedback for us?