

Section 08: Relations, DFAs

1. Relations

- (a) Draw the transitive-reflexive closure of $\{(1, 2), (2, 3), (3, 4)\}$.
- (b) Suppose that R is reflexive. Prove that $R \subseteq R^2$.
- (c) Consider the relation $R = \{(x, y) : x = y+1\}$ on \mathbb{N} . Is R reflexive? Transitive? Symmetric? Anti-symmetric?
- (d) Consider the relation $S = \{(x, y) : x^2 = y^2\}$ on \mathbb{R} . Prove that S is reflexive, transitive, and symmetric.

2. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1, 2, 3\}$.

- (a) All binary strings.
- (b) All strings whose digits sum to an even number.
- (c) All strings whose digits sum to an odd number.

3. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

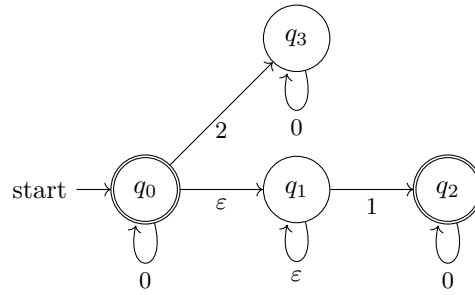
- (a) All strings which do not contain the substring 101.
- (b) All strings containing at least two 0's and at most one 1.
- (c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

4. Relations and Strings

Let $\Sigma = \{0, 1\}$ and define the relation \diamond on Σ^* by $x \diamond y$ if and only if the length of xy is even. (Here $x \diamond y$ is another way of writing $(x, y) \in \diamond$.) Prove that \diamond is reflexive, symmetric, and transitive.

5. NFAs

(a) What language does the following NFA accept?



(b) Create an NFA for the language “all binary strings that have a 1 as one of the last three digits”.

6. DFAs & Minimization

(a) Convert the NFA from 1a to a DFA, then minimize it.

(b) Minimize the following DFA:

